Journal of Economic Interaction and Coordination
https://doi.org/10.1007/s11403-024-00409-z

REGULAR ARTICLE

Transient dynamics of the COVID lockdown on India's production network

Antoine Mandel[1] · **Vipin P. Veetil**[2]

Received: 21 October 2021 / Accepted: 21 February 2024

Abstract

In the wake of the COVID-19 pandemic, the Government of India imposed production restrictions on various sectors of the economy. Prima facie there is reason to believe that the cost of the quantity constraints may be greater than their simple sum. This is because quantity constraints percolate through the production network forcing some sectors to reduce output because of the non-availability of inputs. This paper uses an input–output network model (IO-NET model) to study the impact of the lockdown on the Indian economy. We calibrate our IO-NET model to the Indian economy using data on sectoral linkages. We then examine the impact of the lockdown using sector-based computational experiments. Such experiments allow us to examine the out-of-equilibrium time dynamics that emerge in response to the lockdown. The transient dynamics reveal certain counterintuitive phenomena. The first of which is that the supply of output of some sectors increases during and immediately after the lockdown. Second, recovery after the relaxation of the lockdown entails the overshooting of GDP above its normal levels. And the size of the overshooting depends on the stickiness of prices. These counterintuitive phenomena are intimately related to the network interaction between firms as buyers and sellers of intermediate inputs. The paper also measures the network effect of the lockdown across different sectors. There is sizeable heterogeneity among sectors in how their network position amplifies the quantity constraints imposed on sectors distantly related to them as buyers–sellers of intermediate inputs. Ultimately, models like our own can serve as testbeds for policy experiments, especially when the model is calibrated to granular data on buyer–seller linkages in the economy.

Keywords COVID lockdown · Indian economy · Production network · Disequilibrium · Sector-based model

✉ Vipin P. Veetil
vipin.veetil@gmail.com

1 Paris School of Economics, Université Paris 1 Pantheon-Sorbonne, Maison des Sciences Économiques, 106-112, Boulevard de l'hôpital, 75647 Paris Cedex 13, France

2 Economics Area, Indian Institute of Management Kozhikode, Kozhikode 673570, India

Published online: 19 March 2024

JEL Classification D50 · D57

1 Introduction

Toward the end of March 2020, the Government of India imposed a series of curbs on economic activity to limit the spread of the COVID-19 virus. These measures were perhaps the most pronounced of the responses to the pandemic in terms of their geographic and economic coverage. India is after all the world's third largest economy with nearly half a billion workers. India's comprehensive lockdown, which lasted for at least 9 weeks, therefore constitutes one of the greatest economic experiments in recorded history. While data on the economic impact of the lockdown have become readily available both with respect to aggregate and sectoral output, it remains unclear as to what kinds of models would be most suitable to make sense of the data. Prima facie, workhorse macroeconomic models appear unsuitable to understand the economic dynamics of the COVID lockdown. Many workhorse models assume equilibrium, with some allowing for equilibrium with nonzero excess demands in some markets. It would, however, be difficult to argue that the sudden lockdown of the economy did not create a situation of disequilibrium which may have lasted for a sizeable period of time.

Yet another limitation of standard macroeconomic models in studying the COVID lockdowns is that they tend to assume away the flows of intermediate inputs between firms. The conventional macroeconomic toolbox collapses the economic system to a representative agent, thereby discarding the flows of goods from one sector to another. In reality, a great deal of economic activity takes the form of the flow of intermediate inputs from one firm to another. These flows are the means by which the impact of the lockdown on one sector is transferred to another. Some firms have had to reduce output simply because of the non-availability of inputs or a decrease in demand for its output. The lockdown shock therefore travelled upstream and downstream through the production network. Ignoring the flows of intermediary inputs means assuming away one of the primary mechanisms by which the shock percolated through the economic system.

What we need therefore is a model which allows for the disequilibrium propagation of quantity constraints through the production network. In this paper, we present one such model. Note that equilibrium within a network economy is similar to general equilibrium in the Arrow–Debreu sense with the addition that the flows between firms/sectors must be invariant over time steps. Acemoglu et al. (2012) present this definition of equilibrium in a network economy. Gualdi and Mandel (2016) generalize Acemoglu et al.'s model by allowing for out-of-equilibrium interactions in two specific ways, the first of which is that the flows between firms/sectors can vary from one time step to the next, the second of which is that firms can enter–exit the network. In this paper, we use a simplified version of Gualdi and Mandel's model in which there is no entry–exit of firms; therefore, the only disequilibrium that can occur is a variation in the flows between firms/sectors from one time step to the next. Opening up to disequilibrium analysis means a variety of modeling choices must be made. These include choices about how firms/sectors set prices, how they determine quantities, how they form expectations about the future, what happens to stock of unsold goods, and what rules determine the allocation of goods in case of positive excess demand. Note that given our modeling choices along these margins, the model economy converges to equilibrium starting from random initial conditions. In fact, convergence to equilibrium is guaranteed by a theorem from the field of Random Matrices, which requires that the network be aperiodic and irreducible. Therefore our model converges to equilibrium for a variety of specifications along these out-of-equilibrium margins, including different specifications of the stickiness of prices. The model therefore ingrains local and global stability. This means that all we are studying in response to the COVID lockdown shock is how aggregate and sectoral output is affected as the system is shocked far from equilibrium and then slowly works its way to equilibrium over time.

The specifics of our model are as follows. There are a finite number of firms related to each other as buyer–sellers of intermediate inputs, with each firm producing a unique good using a Cobb–Douglas production function. Each firm determines its price using information about the demand for its input and the existing quantity of output. Price determination therefore occurs through a local non-tatonnement process. Firms purchase inputs using revenues earned at the last time step. The output produced from these inputs are carried over and sold at the next time step. Our model therefore ingrains a temporal relation between production and pricing decisions. More specifically, revenues earned at $t-1$ influence prices at $t+1$. In so far as inputs must be purchased using in-hand liquidity, firms can be thought of as facing cash-in-advance constraints. The model does not contain banks or other financial institutions. We close the model with a representative household who supplies a fixed quantity of labor to all firms and purchases goods using a Cobb–Douglas utility function. With these features, our model differs from the standard input–output model in that it allows for the reallocation of labor and intermediate inputs in response to changes in relative prices. Furthermore, unlike most variants of the input–output setting, our model allows for the carrying of inventory in response to negative excess demands. It also allows for the carrying of cash reserves in so far as quantity constraints prevent the use of all available liquidity to purchase inputs. The standard input–output system does not allow for arbitrary shocks to arbitrary sectors given a fixed network because there must be consistency between the inputs and outputs. The model used in this paper allows for arbitrary shocks because we impose neither equilibrium flows, nor market clearing.[1]

We implement the COVID lockdown as quantity constraints in the following form. Firms are not allowed to produce more than a prescribed quantity. Firms

[1] Note that our model embeds a coordination problem that is subtly different from the equilibrium coordination problem models described by Foley. Unlike the social coordination problem described by Foley where each agent cares about the decisions of all others captured by some aggregate variable, our model studies a social coordination problem that emerges from the fact that each agent makes decisions based on the decisions of a small set of agents from the population of all agents. More specifically, each firm/sector within our model is directly influenced only the prices set by its input seller and the demands given by its output buyers. The coordination problem arises because of the web of inter-relations between the decisions of many agents, each tied to a few other agents, thereby through long chains related to everyone in the economic system.

accordingly reduce the supply of output and the demand for intermediate inputs. Limits on the supply of output generate inventory and limits of demand for intermediate inputs generate cash reserves. The lockdown shock travels upstream and downstream through the production network. As the shock propagates, the economy goes through a period of disequilibrium in which some firms produce less than the quantity constraints imposed by the lockdown simply because of the non-availability of inputs. The cost of the lockdown therefore can be sizeably greater than the simple sum of the quantity constraints imposed on each sector. With the exact size of the cost determined by the specifics of the flows of intermediate inputs and other model parameters.

We calibrate the model to the Indian economy using the most recent input–output table representing each sector with one firm. With a Cobb–Douglas production function, the optimal share of firm *i*'s expenditure on the input supplied by firm *j* does not depend on the prices. Rather the shares are determined by the exponents of the Cobb–Douglas production function. We use the weights of the flows between sectors in the input–output table to determine the Cobb–Douglas exponents. In so far as the Table is consistent, the calibrated version of our model maintains the empirically observed relative sizes and relative flows between sectors.

We build two scenarios of the sectoral composition of the quantity constraints associated with the lockdown based on the scenarios created for European countries by the IFO-Institute (2020). We then impose these quantity constraints within our sector-based model to study the time series response of the output of different sectors to the lockdown itself and the relaxation of the lockdown. According to our simulations, at its lowest point, the GDP is less than half of that which would be generated by a simple sum of the quantity constraints on each sector. Put differently, at its worst, the production network amplifies the direct impact of the sectoral quantity constraints by two folds.

1.1 Related literature

Our approach to modeling the economic system echoes some of Lance Taylor's concerns about modeling macroeconomic dynamics of low-income countries, mostly notably expressed in his book "Structuralist Macroeconomics: Applicable Models for the Third World". Taylor (1983) argued that economic dynamics is influenced by specific durable features of the economic system, some of which can neither be deduced nor seamlessly altered by profit maximizing behavior of firms. One of these features is the buyer–seller network between firms in an economy, which emerges out of a long history of interactions between firms, with many of these interactions having to do with historically specific circumstances. We incorporated these "structures" into our analysis much like one would incorporate primitives about economic actors themselves. In this sense, our work is considerably more 'structuralist' than workhorse macro models. We also share Taylor's view that some sectors clear via quantity adjustments rather than price adjustments. In fact, much of the dynamics of our model during the lockdown is generated by the fact that price adjustment do not clear markets, and therefore firms must deal with the problem of positive and negative excess demands. Quantity adjustment are the primary drivers of the dynamics of our model.

Taylor was very cognizant of the fact that workhorse macro models 'get it wrong', and that they get it particularly wrong when it comes to low-income countries. Structure matters more in low-income countries because they are often characterized by under-developed markets and have a shorter history of 'capitalist economic growth'. With less developed markets (financial and real), a sizeable share of economic activity is coordinated by non-price mechanisms and non-contractual relations between individuals. Much of the structure of interactions between individuals and groups is inherited from a feudal past, capitalism as it were is merely the top-soil. What this means is that all of what determines economic outcomes cannot be deduced from history-independent primitives about the behavior of individuals. Structures that have emerged from the interactions of the past in a wholly different setting continue to shape economic outcomes at all levels from the granular to the macro. While the use of workhorse macro models can be justified in the context of high-income economy with the argument that their structures have been formed by capitalist economic growth, and therefore can be in principle deduced from profit-maximizing behavior, such an argument holds no water with respect to low-income countries. Put differently, not all of what is necessary for an economic analysis of most low-income countries can be deduced by primitives about individuals or firms, there are structures from the past that influence the present which have emerged from behaviors that have little to do with doing the best for oneself within a market setting. We are tempted to say one must account for the long shadow of the past, except for the fact that in many cases the present is the shadow while the past is the real substratum that is driving economic dynamics.

Our model incorporates some features of the disequilibrium models developed by Edmond Malinvaud in the 1970s and early 1980s. Malinvaud (1977) offered a fixed price model of macroeconomic dynamics. Note that within our model too prices are fixed during the lockdown phase, which allows us to focus on quantity adjustments. We extend on the fixed price idea by allowing for varying flexibility in prices after the lockdown ends. Malinvaud (1982) specified an n-sector model to study out-of-equilibrium dynamics within a high dimensional system. In some senses our model extends Malinvaud's specification by allowing for the demands between sectors to directly influence each other's production. In his lecture at the Fourth World Congress of the Econometric Society, Malinvaud (1981, p. 1368) says that several models take into account disequilibria concerning quantities "but price disequilibria do not have the place which should be theirs". Our model allows for the simultaneous study of price and quantity disequilibria. In fact, one of the very interesting features of our model is the fact that local market-clearing prices do not necessarily guarantee equilibrium. Put differently, a system with flows between sectors/firms allows for disequilibrium with zero excess demand in all markets. This is because there are multiple sets of prices that clear all markets but not all of these are general equilibrium prices. Disequilibrium with zero excess demand in all markets is characterized by changes in the sizes of sectors/firms from one time-step to the next, or equivalently changes in the flows from one sector to another. General equilibrium prices within our setting are characterized by unchanging flows between sectors.

In some senses, this paper is an application of our previous work on the COVID lockdowns to the Indian economy. In Mandel and Veetil (2020b), we study the cost of supply chain disruptions using the world input–output table, while in Mandel and Veetil (2020a), we study the impact of the lockdowns on the US economy using a granular data set on buyer–seller relations between firms. One of the theoretical contributions which distinguishes this paper from our previous work on the COVID lockdowns is the idea of 'an overshooting recovery' and its relation to price stickiness. More specifically, the buildup of inventory during the lockdown and its immediate aftermath generates an increase in GDP at a later date as the inventory finds its way first into the market for intermediate inputs and then final goods. The time sequence and size of the overshooting depend on the degree of price-stickiness because it mediates the relation between changes in demand and supply, on the one hand, and inventory buildup on the other.

Numerous economists before us have used variants of the standard input–output model to study the impact of the COVID lockdowns, with applications to Columbia, Germany, Ireland, Italy, and the UK among other countries.[2] Some studies have focused on the propagation of the shock via restrictions on final demand, whereas others have emphasized the differential impact of labor supply across sectors (Osotimehin and Popov 2020; Barrot et al. 2020). Our analysis in contrast emphasizes the propagations of restrictions in the availability of intermediate inputs. Inoue and Todo (2020) and Inoue et al. (2021) report results from simulations based on granular buyer–seller firm level data for Japan. While our model is capable of using large granular buyer–seller network data, we have had to use India's input–output table instead because of the absence of sizeable granular data on buyer–seller relations between firms in India.

Some economists have attempted to study the COVID lockdowns using equilibrium models, see, for instance, Baqaee and Farhi (2020). While equilibrium models enable the derivation of closed-form solutions, they assume away the possibility of relative price deviations and temporary resource-reallocations generated by the lockdowns.[3] Within the equilibrium-setting, the production network is little more than a means to amplify the shock. In fact, as Baqaee and Farhi (2020) show, within an equilibrium setting, the network can be wholly dispensed with information on relative shares of inputs. Naturally, such a dispensing of economic networks cannot happen in a system capable of exhibiting disequilibrium dynamics. Perhaps the most distinguishing theoretical aspect of our model is that it allows for temporary reallocations of resources in response to the lockdowns. Put differently, from a theoretical

[2] For studies using input–output tables to under the impact of the COVID lockdowns on other economies, see Bonet-Morón et al. (2020), Fadinger and Schymik (2020), McCann and Myers (2020), Giammetti et al. (2020) and Richiardi et al. (2020).

[3] From an empirical point of view, one of the primary shortcomings of equilibrium models of COVID lockdowns is their inability to generate the sizeable fluctuations in sectoral and subsectoral outputs, along with their complex nonlinear time dynamics. Figure 1 of the paper presents these complex time dynamics of subsectoral levels for the Indian economy. Our model is able to generate some aspects of the nonlinear time dynamics of sectoral outputs, with the outputs of some sectors rising in response to the lockdown shock as observed in the data (Fig. 4). See the following papers for an equilibrium treatment of the COVID lockdown shocks.

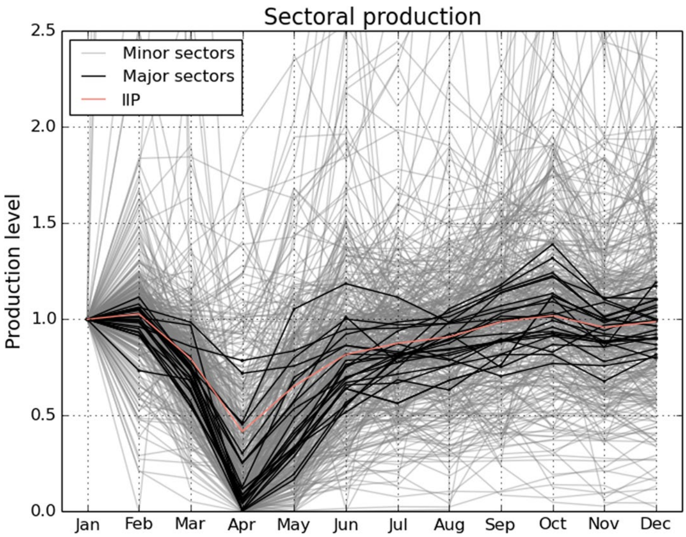


Fig. 1 Time series of Index of Industrial Production, minor sectors, and major sectors

point of view our model allows for the study of the explicit time sequence of events rather than mere comparative statics among equilibrium configurations. Ultimately the choice between the two sets of models—equilibrium and disequilibrium—is an empirical question. Does data suggest that the economy deviated sufficiently from equilibrium in response to the COVID lockdown? Naturally, an empirical investigation of this question is beyond the scope of the present paper. But elementary empirical evidence does suggest that the question is worth pursuing. Consider, for instance, Fig. 1, which presents the time series of output at three levels of aggregation: the Index of Industrial Production, major sector (NIC 2 digit), and minor sector (NIC 6 digit). The data are de-seasonalized and then normalized to production level in the month of January 2020. One would be hard pressed to argue that such data can be generated by an economy which is perennially at equilibrium, or jumping from one equilibrium to another. The data suggests that there has been sizeable reallocation of resources at the minor sector level since the COVID lockdowns. The smooth convergence of the Index of Industrial Production to the pre-COVID level is a consequence of massive reallocation of resources at the granular level. To us, these reallocations signal disequilibrium dynamics.

While there is sizeable India specific work on the pandemic, few studies consider the economic impact of the pandemic on the economy as a whole rather than on particular sectors or on firms of particular sizes.[4] This is important because the

[4] It is worth mentioning some of the India specific papers. Dev and Sengupta (2020) note the state of the Indian economy before the pandemic began and the constraints on policy options available to respond to it. While Goyal (2020) discusses possible policy responses to the COVID lockdown using old style simple aggregate Keynesian thinking. Kanitkar (2020) studies the impact on the energy sector, Sahoo and Ashwani. (2020) note the impact on MSME and trade, and Mamgain (2021) examines the effect on the labor market. Lastly, Sengupta (2020) studies the impact on output as a decline in labor today reduces

effect on the economy as a whole is not merely a simple sum of partial effects. The handful of studies that do consider the impact on the Indian economy as a whole use aggregative models that do not incorporate supply chain dynamics. One study worth mentioning is that of Mahajan and Tomar (2021). They empirically measure the disruption generated by the lockdowns on India's agricultural supply chain. Their study suggests that the COVID restrictions generated sizeable supply chain disruptions and relative price deviations.

To summarize, our paper has several theoretical and empirical novel elements. The first novel element has to do with an application of a network model to studying the out-of-equilibrium dynamics that follow from quantitative restrictions like the COVID lockdown.[5] Our benchmark in this sense is not only the representative-agent DSGE model but the class of DSGE models more generally. This is because our model involves a system in which prices are set by firms using local information on demand and supply. This creates a social coordination problem wherein though general equilibrium prices must reflect global variables, firms/sectors decisions are based on local variables. And there is no external agent to solve this coordination problem. The system as it were "solves" the problem through a bottom-up process of local decision-making. The second novel contribution is the discovery of the surprising relationship between the price-stickiness and the impact of a real shock on the economic system. Within most workhorse macroeconomic models, price-stickiness accentuates the impact of real shocks on the economic system. Within our network economy model, price-stickiness (within reasonable bounds) can dampen the impact of real shocks. The third novel element has to do with explaining the over overshooting of GDP of the Indian economy after the lifting of the COVID lockdown shock. Note the time dynamics of the overshooting is intricately related to the stickiness of prices. The fourth novel element has to do with explaining the increase in the output of some sectors during the lockdown period. This explanation depends vitally on the out-of-equilibrium nature of dynamics that unfold in response to the COVID lockdown.

1.2 Organization of the paper

The paper is organized as follows. Section 2 describes the model. It explicitly enumerates the time sequence of events that generate out-of-equilibrium dynamics. The section summarizes the calibration of the model to India's input–output table. It also notes the conditions for the existence and stability of equilibrium within our model. Section 3 describes the setup of the computational experiments and uses the data so generated to analyze the transient time dynamics of aggregate and sectoral output. Section 4

Footnote 4 (continued)

capital formation and therefore future output. And Vidya and Prabheesh (2020) study the impact of the pandemic on the global trade network with particular emphasis on India.

[5] The closet models to our own in this sense are Inoue and Todo's (2020) account of the Japanese economy and Asian Development Bank's MIROT model. While these models do not assume equilibrium, nor do they explicitly study the out-of-equilibrium dynamics that emerge when firms/sectors respond to quantity constraints.

computes the network multiple for different sectors of the economy. The 'network multiple' is a measure of the amplification of the quantity constraint by the topology of the production network. The section also relates the sectoral network multiples to the position of the sectors within the supply chain. Section 5 explains the phenomena of 'overshooting recovery' and its relation to price-stickiness. Section 6 presents concluding thoughts. The model code written in Python programming language is available at bitbucket.org/VipinVeetil/network economy.

2 The model

This section recalls the workings of our input–output network model denoted by the acronym IO-Net model. The full version of IO-Net model allows for firm entry and exists from the production network along with decision-making on prices, quantities, and network weights. In this paper, however, we use a version of IO-Net model in which the production network is fixed, though firms do make decisions on prices and quantities. IO-Net model has been used to study the emergence of scale-free production networks (Gualdi and Mandel 2016), the relation between network topology and innovation in generating economic growth (Gualdi and Mandel 2018), and the behavior of prices in response to monetary shocks (Mandel et al. 2019; Mandel and Veetil 2021). The model has a bottom-up veneer as firms make decisions using local information about demands and input prices. Since the model does not impose equilibrium, it allows us to explicitly study the transient dynamics that emerge from endogenous and exogenous disturbances.

2.1 Basic setup

The economy is populated by a finite set of firms and a representative household. $N = \{1, \dots, n\}$ denotes the set of firms and the set of goods (as each firm produces a unique good). The representative household is indexed by 0. The household supplies an invariant quantity of labor l normalized to 1. The household has Cobb–Douglas preferences:

$$u(x_1, \dots, x_n) = \prod_{i=1}^{n} x_i^{\beta_i} \tag{1}$$

with $\sum_{i=1}^{n} \beta_i = 1$ and $\beta_i > 0$ for all $i \in N$. In other words, the household purchases a positive quantity of goods from each firm in the economy. The firms interact through a production network. More specifically, each firm i has a production function $f_i : \mathbb{R}_+^M \to \mathbb{R}_+$ of the form

$$f_i(l_i, (y_{ij})_{j \in N}) = l_i^{a_{i0}} \left(\prod_{j \in N} y_{ij}^{a_{ij}} \right)^{(1-a_{i0})} \tag{2}$$

where $l_i \in \mathbb{R}_+$ is the labor input, $y_{ij} \in \mathbb{R}_+$ the input of good j, a_{i0} is the share of labor and $a_{ij} \in \mathbb{R}_+$ is the non-labor share of firm i's production expense spent on the input from firm j. We thus assume $\sum_{j \in N} a_{ij} = 1 - a_{i0}$ for all $i \in N$. The matrix $A = (a_{ij})_{i,j \in N} \in \mathbb{R}_+^{N \times N}$ is the weighted adjacency matrix of the production network of the economy, with $a_{ij} > 0$ if and only if j is a supplier of input to i. Let y_{ij} denote the flow of goods from j to i if $a_{ij} > 0$. Let $\mathcal{E}(A)$ denote the economy characterized by this network. The general equilibrium of $\mathcal{E}(A)$ is defined as follows:

Definition 1 A general equilibrium of the economy $\mathcal{E}(A)$ is a collection of prices $(\overline{p}_1, \ldots, \overline{p}_n) \in \mathbb{R}_+^N$, wage $\overline{p}_0 \in \mathbb{R}_+$, production levels $(\overline{q}_1, \ldots, \overline{q}_n) \in \mathbb{R}_+^N$, consumption levels $(\overline{x}_1, \ldots, \overline{x}_n) \in \mathbb{R}_+^N$, labor $\{\overline{l}_i\}_{i \in N} \in \mathbb{R}_+^N$ and commodity flows $\{\bar{y}_{ij}\}_{i,j \in N} \in \mathbb{R}_+^{N \times N}$ such that:

1. Markets clear:

$$\forall i \in N,\ \overline{q}_i = \overline{x}_i + \sum_{j=1}^{n} \bar{y}_{ji}, \forall i \in N \text{ (goods market)} \tag{3}$$

$$1 = \sum_{i=1}^{n} \overline{l}_i \text{ (labor market)} \tag{4}$$

2. The representative consumer maximizes utility: $(\overline{x}_i)_{i \in N}$ is a solution to

$$\begin{cases} \max & u(x_1, \ldots, x_n) \\ \text{s.t} & \sum_{i=1}^{n} \overline{p}_i x_i \leq \overline{p}_0 \end{cases} \tag{5}$$

3. Firms maximize profits: for all $i \in N, (\overline{q}_i, \overline{l}_i, (\bar{y}_{ij}) \in \mathbb{R}_+^{N+2}$ is a solution to

$$\begin{cases} \max & \overline{p}_i q_i - \overline{p}_0 l_i - \sum_{j \in N} \overline{p}_j y_{ij} \\ \text{s.t} & q_i \leq f_i(l_i, (y_{ij})_{j \in N}) \end{cases} \tag{6}$$

Note that the definition is standard, except for commodity flows between firms. The proof of existence of equilibrium of such an economy uses certain results from random matrix theory. More specifically, the existence of a representative household, which buys goods from all firms and sells labor to all firms, implies that the network is aperiodic and irreducible (Livan and Novaes 2017). Aperiodicity and irreducibility of the matrix describing the economy guarantee the existence of a unique equilibrium upto price normalization, where by 'upto price normalization', we mean the fact that the price level of the economy is determined by the quantity of money, while the relative prices characterizing the equilibrium are determined by real primitives.

Remark 2 Within a network economy, the aperiodicity and irreducibility of the network guarantees the existence of equilibrium. The network is irreducible if starting from any firm (or the household) it is possible to reach any other firm (or the household). The network is aperiodic if the greatest common divisor of its cycle lengths is one. The existence of a representative household who buys goods from all firms and sells labor to all firms ensures aperiodicity and irreducibility. For details on the existence of equilibrium of the network economy presented in this paper, see Assumption 1 and Proposition 1 of Mandel and Veetil (2021).

2.2 Out-of-equilibrium dynamics

Time is discrete and indexed by $t \in \mathbb{N}$. Each firm $i \in N$ is initially endowed with a random amount of working capital $w_i^0 \in \mathbb{R}_+$ and a random amount of output $q_i^0 \in \mathbb{R}_+$. All random amounts are drawn from a *uniform*(0, 1) distribution. Each agent then engages every period in a sequence of local interactions with its connections (buyers and sellers) in the production network. More specifically, the following sequence of events take place every time step $t \in \mathbb{N}$:

1. Agents determine nominal demand to their suppliers according to network weights: the nominal demand of firm i toward firm j is $a_{ij}w_i^t$. The nominal demand of the household toward firm j is given by $\beta_j w_0^t$. And the nominal demand of firm i for labor is $a_{i0}w_i^t$.
2. Firm j sets the price $p_j^t \in \mathbb{R}_+$ at its "local" market-clearing value:

$$p_j^t = \frac{\sum_{i \in N} a_{ij}w_i^t + \beta_j w_0^t}{q_j^{t-1}} \tag{7}$$

The household sets market-clearing wage:

$$p_0^t = \sum_{i \in N} a_{i0}w_i^t \tag{8}$$

Remark 3 Note that local market clearing prices do not imply that the economy is in equilibrium. The agents use only information about their own supply and demand to set prices. The network economy can be out of equilibrium with all markets clearing if prices do not correspond to general equilibrium flows.

3. Goods flow proportionally to demand, for all $i, j \in N$:

$$y_{ij}^t = \frac{a_{ij}w_i^t}{p_j^t} \text{ (inputs allocation)} \tag{9}$$

$$x_i^t = \frac{\beta_i w_0^t}{p_i^t} \text{ (consumption allocation)} \tag{10}$$

$$l_i^t = \frac{w_i^t}{\sum_{j\in N} w_j^t} \text{ (labor allocation)} \tag{11}$$

4. Firms update their working capital based on revenues, for all $i \in N$:

$$w_i^{t+1} = \sum_{j\in N} a_{ji} w_j^t \tag{12}$$

And the household's wealth w_0^{t+1} is simply the wage p_0^t because quantity of labor is normalized to 1.

5. Firms produce output for the next period (and labor supply is replenished to 1). Namely, for all $i \in N$:

$$q_i^t = f_i(l_i^t, y_{ij}^t) \tag{13}$$

2.3 Calibration

We calibrate the model using India's input–output table for the year 2017 (ADB 2020). We include 34 sectors from the Table. We exclude international trade, government, and capital formation. The representative household's outflow of money to each sector is given by the personal consumption expenditure of households. The representative household's inflow of money from each sector is set equal to 2/3 of the value added by the sector. This is to understand the standard assumption that labor accounts for about 2/3 of national income. The exclusion of certain sectors from the input–output table generates an imbalance between the inflows and outflows of some sectors. These imbalances are rectified by creating a 'balancing sector', which has inflow and outflows with different sectors so as to balance the input–output table. Finally, the balancing sector is merged with the household sector to create a grand-household sector which has inflows and outflows with all sectors in the economy. We use the weights of the input–output table to compute the weights of the production network. More specifically, we set the a_{i0} as the weight of the flow of money from each sector i to the household. And we compute a_{ij} using the weights of the flow of money from sector i to sector j. We set β_i to equal the weight of the household's expenditure on sector i. This completes the calibration of the model to the Indian economy at the sectoral-level.

2.4 Convergence to equilibrium

We measure the convergence of the calibrated model to equilibrium using mean absolute price change $\delta^t = \frac{1}{n}\sum_{i=1}^n \frac{|p_i^t - p_i^{t-1}|}{p_i^{t-1}}$, where p_i^t is the price of firm i at time step t. Figure 2 shows the time series of δ^t from a model simulation. Mean absolute price change δ^t decreases to below 10^{-14}, thereby indicating that the economy has converged to the

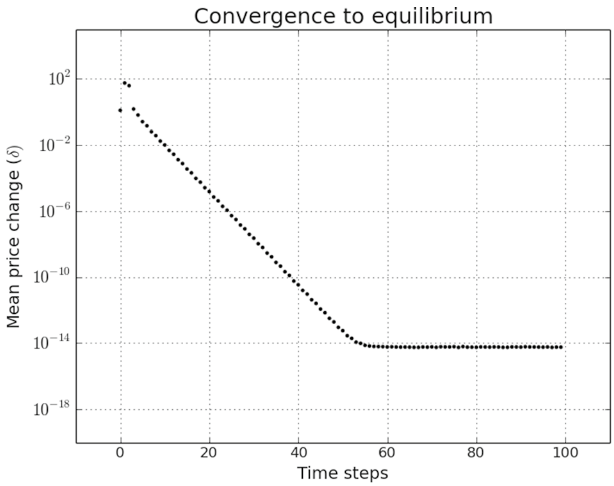


Fig. 2 Convergence of the model to equilibrium

neighborhood of equilibrium. Note that due to numerical reasons δ^t will never reach the value of zero.[6] In the next section, we introduce lockdown policy within a setting in which the economy has reached the neighborhood of equilibrium (i.e., $\delta^t < 10^{-14}$).

3 Computational experiments with lockdown policy

An agent-based model is essentially a synthetic economy in silico. Such models consist of agents—who may be households, firms, traders, or other economic entities—and an interaction environment. Agents are characterized by rules of behavior using which they interact with each other. The data which emerges from such interactions are collected and analyzed to understand model dynamics.[7] There are two types of agents within our model: sectors and the representative household.[8] Most agent-based models are populated with agents that either firms or individuals rather than sectors. Our model is capable of being specified at the level of individual firms. Unfortunately, firm buyer–seller network data is not available for the Indian economy. And therefore, we have had to specify our model at the level of sectors. In this sense, the model presented in this paper may be thought of more as a sector-based model rather than a truly agent-based model. It must, however, be added that

[6] For more simulation results on the model's convergence to equilibrium see Mandel et al. (2019, pp. 9–10). For results on theoretical bounds of the convergence to equilibrium see Mandel and Veetil (2021, Lemma 1 and Proposition 2).

[7] See Borrill and Tesfatsion (2011) and Axtell et al. (2000) for an introduction to agent-based models. See Epstein (1999) for a discussion about how the 'generative' approach ingrained in agent-based models is distinct from both the deductive and inductive methods. And see Arthur (2006) for a discourse on how agent-based models can be used to study out-of-equilibrium dynamics.

[8] The representative household is an analytical simplification that allows us to focus on the macroeconomic consequences of inter-sectoral flows.

despite the less granular specification, the model is used to study out-of-equilibrium dynamics much like other agent-based models..

Note that we implement only the first set of nationwide lockdowns which lasted from 25 March to 31 May. This is because of the paucity of data. More specifically, from June began a series of relaxations of the lockdown along with the delegation of responsibilities to state governments. There are, however, little data on the exact implementation of the lockdown by sectors across different regions of the country after June. These data restrictions mean that our paper merely illustrates the usefulness of the network perspective in understanding some key dynamics that follow from quantity constraints. We make no attempt to match the empirical magnitude of the changes in GDP in 2020 and 2021. Our model can indeed be used to perform such an exercise. However, that would require two sets of data which we do not presently possess. First, granular data on buyer–seller relations between the millions of firms that populate the Indian economy. Second, temporal data on the quantity constraints imposed on different sectors by the central and state governments. In the absence of these two sets of data, what we have attempted to do in this paper is to provide a 'proof of concept' of the usefulness of an agent-based model of a network economy in understanding the consequences of policy action.

The rest of this section is organized as follows. Section 3.1 tabulates the quantity constraints on different sectors. Section 3.2 explains how the lockdown constraints are implemented within the model. Section 3.3 analyzes the model data to describe the transient dynamics that emerge in response to the lockdown.

3.1 The scenarios

We develop two scenarios based on the size of the direct impact of the lockdown on different sectors of the economy. These scenarios are based on the scenarios created for European countries by the IFO-Institute (2020), with minor alterations for some sectors.[9] The sectors themselves come from the classifications in the input–output table provided by the Asian Development Bank (2020). Table 1 lists the sectoral codes in the input–output table, the names of the sectors, and the quantity constraints under Scenario A and Scenario B. The figures under the two Scenarios mark the quantity constraint imposed on different sectors during the lockdown as a proportion of their normal output. For example, a figure of 0.3 for sector i implies that during the lockdown period sector i's output level cannot exceed 30% of its normal output. The lockdown is more severe under Scenario B than Scenario A.

3.2 The lockdown constraints

During the lockdown, firms are forced to produce no more than the ratio of their lockdown output to steady-state output. More specifically, let b_i denote firm i's

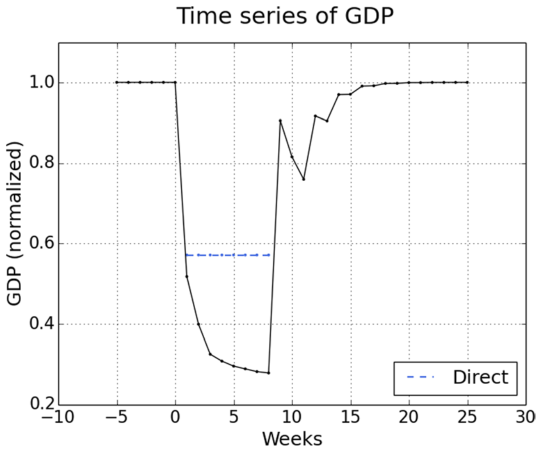
[9] The following modification were made to IFO Scenario 1, the IFO lockdown production in brackets preceded by our Scenario A: 'Coke and refined petroleum products' 0.5 [IFO 0.2], 'Sale, maintenance, and repair of motor vehicles and motorcycles; retail sale of fuel' 0.5 [IFO 0.2], 'Post and telecommunications' 0.8 [IFO 0.2], 'Education' 0.5 [IFO 1].

Table 1 Sectoral output during lockdown as a proportion of normal output

Code	Sector	Scenario A	Scenario B
c1	Agriculture, hunting, forestry, and fishing	1	0.8
c2	Mining and quarrying	0.5	0.4
c3	Food, beverages, and tobacco	1	0.8
c4	Textiles and textile products	0.2	0.1
c5	Leather, leather products, and footwear	0.2	0.1
c6	Wood and products of wood and cork	0.2	0.1
c7	Pulp, paper, paper products, printing, and publishing	0.2	0.1
c8	Coke, refined petroleum, and nuclear fuel	0.5	0.4
c9	Chemicals and chemical products	0.2	0.1
c10	Rubber and plastics	0.2	0.1
c11	Other nonmetallic minerals	0.2	0.1
c12	Basic metals and fabricated metal	0.2	0.1
c13	Machinery, nec	0.2	0.1
c14	Electrical and optical equipment	0.2	0.1
c15	Transport equipment	0.2	0.1
c16	Manufacturing, nec; recycling	0.2	0.1
c17	Electricity, gas, and water supply	1	1
c18	Construction	0.5	0.1
c19	Sale, maintenance, and repair of motor vehicles and motorcycles; retail sale of fuel	0.5	0.1
c20	Wholesale trade and commission trade, except of motor vehicles and motorcycles	0.5	0.5
c21	Retail trade, except of motor vehicles and motorcycles; repair of household goods	0.5	0.5
c22	Hotels and restaurants	0.2	0.1
c23	Inland transport	0.2	0.1
c24	Water transport	0.2	0.1
c25	Air transport	0.2	0.1
c26	Other supporting and auxiliary transport activities; activities of travel agencies	0.2	0.1
c27	Post and telecommunications	0.8	0.5
c28	Financial intermediation	0.5	0.5
c29	Real estate activities	0.5	0.2
c30	Renting of M &Eq and other business activities	0.2	0.1
c32	Education	0.5	0.2
c33	Health and social work	1	1
c34	Other community, social, and personal services	0.5	0.5

lockdown output as a proportion of steady-state output, x_{ji} the real quantity of input j bought by firm i in steady state, and d_{ij} the nominal demand for input j of firm i in steady state. The lockdown constraints take the following form. On the supply side, the quantity of intermediate input j bought by firm i is constrained by an

Fig. 3 Time series of GDP after the Lockdown



upper bound of $b_i x_{ji}$. On the demand side, the nominal demand of firm i for input j is constrained by an upper bound of $b_i d_{ij}$. Naturally, these lockdown constraints are capable of generating inventory dynamics and cash reserve dynamics. Each firm treats its inventory carried over from the previous time step no different from the output produced at the current time step. Similarly, each firm treats the cash reserves carried from the previous time step no different from the sales receipts earned at the present time step.

3.3 Transient dynamics induced the lockdown

Figure 3 shows the time series of GDP after a 9 week lockdown under Scenario A. The lockdown begins at time step zero. The dotted blue line marks GDP computed by scaling the output of each sector by the quantity constraint noted in Table 1. In other words, the dotted blue line marks the direct cost of the lockdown, i.e., the impact of the lockdown on GDP, in a setting where the sectors do not depend on each other as buyers and sellers of intermediate inputs. Figure 3 shows that the GDP declines significantly below the level denoted by the direct impact of the lockdown. In fact, at its lowest, the GDP is nearly half of the level computed using a simple weighted sum of sectoral constraints. The production network therefore amplifies the impact of the lockdown shock. Table 2 enumerates GDP in Quarter 1, Quarter 2, and Quarter 3 of 2020, normalized to their steady-state levels, under Scenario A and Scenario B. Note that the most sizeable impact of the lockdown is in Q2 2020, with the difference between the two scenarios being about ten percentage points.

Figure 4 presents the time dynamics of sectoral output normalized to their steady-state levels. The figure contains all 33 sectors listed in Table 1. Note that while the supply of output of the majority of sectors declines during the lockdown, there are some sectors whose supply of output increases, where by ‘supply of output’ we mean the total output of the firm (representing the sector) which is the sum of the output produced at a time step and the inventory carried over from the previous time

Table 2 Lockdown GDP under Scenarios A and B

Quarter	Scenario A GDP	Scenario B GDP
Q1 2020	0.96	0.95
Q2 2020	0.57	0.47
Q3 2020	1.00	0.99

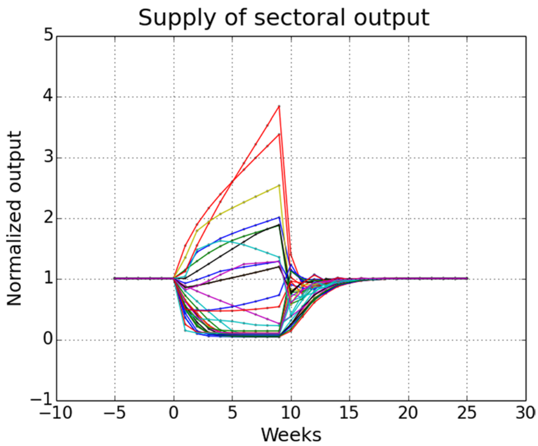


Fig. 4 Time series of output of different sectors of the economy. Each color represents the time series of output of a specific sector. The output of all sectors is normalized to their specific pre-lockdown output

step. Suppose, the quantity constraint imposed on the sector is given by $b_i \in (0, 1)$. The upper bound on the quantity of input j purchased by sector i is given by $b_i x_{ji}$. And therefore, the upper bound on output of sector i is given by $b_i^{1-\alpha} q_i^*$, where q_i^* is the steady-state output of sector i. Suppose sector i produces an output of $q_i^t = b_i^{1-\alpha} q_i^*$ at time step t but its sales are $s_i^t < q_i^t$. In which case an inventory of $I_i^t = s_i^t - q_i^t$ will be carried over to the next time step. The supply of output available at time step $t + 1$ will be $Q_i^{t+1} = q_i^{t+1} + I_i^t$. And it is well possible that $Q_i^t > q_i^*$ (note that in steady state there is zero inventory[10]). When $Q_i^t > q_i^*$, the supply of sector i is above the steady-state level during the lockdown despite the imposition of quantity constraints. As to whether $Q_i^t > q_i^*$ depends on the relation between the changes in demand and changes in the supply of inputs faced by sector i in the time steps preceding t. And these changes depend on the network of relations between sectors and the distribution of quantity constraints imposed on different sectors. Ultimately therefore the topology of the production network determines which sectors exhibit a counter-intuitive increase in supply during the lockdown by shaping the flows of real resources and nominal demands over time.

The sectors that mark the greatest increase as measured by their peak supply of output are 'food, beverages, and tobacco' and 'electricity, gas, and water supply', with the peak supply as a proportion of steady-state output being approximately 3.8 and 3.4, respectively. In fact, sixteen of the thirty-three sectors exhibit a greater

[10] For ease of analysis, we assume that the 'normal' steady-state level of inventory is zero.

than steady-state level of output at some point during the transition from the pre-lockdown equilibrium to the post-lockdown equilibrium. The sectors that exhibit the greatest decline in output are 'rubber and plastics' and 'transport equipment', with the trough of the supply as a proportion of steady-state output being approximately 0.04 for both sectors.

The reason for why the supply of output of some sectors increases beyond their steady-state level during and after the lockdown is as follows. The quantity constraints imposed by the lockdown are heterogeneous across sectors. And the network itself is heterogeneous in terms of the weights of the linkages of each sector to other sectors. This means that the imposition and the relaxation of the lockdown generates sizeable reallocation of resources across different sectors of the economy. Suppose, for instance, sector k sells intermediate inputs to sectors i and j. And the quantity constraint faced by sector i is less than that faced by sector j. In this case, sector j would be able to gather a greater share of input k than at steady state, allowing sector j to expand output beyond the steady-state level. (Sector j may not be able to sell more output than at steady state, but it would be able to carry the output as inventory to future time steps.) These demand side forces have their parallel on the supply side. Suppose sector s purchases inputs from sectors m and n. And the quantity constraint imposed upon sector m is greater than that on sector n. This means sector m's supply of input to sector s will decline more than sector n's supply. Since money flows opposite to the direction of the flow of goods, some of sector m's revenues will be diverted to sector n. This diversion of the revenues can lead to an increase in the nominal size of sector n. The money will find its way toward the purchase of inputs and expansion of output in future time steps, thereby increasing the supply of output of sector n. The demand and supply way of understanding the reallocation of resources is merely a useful analytical device, in reality within a network setting one firm's demand is another firm's supply, and therefore demand and supply forces necessarily operate together. Furthermore, these forces may emanate far away from the firms that end up expanding the supply of output after the lockdown. A firm may be able to expand the supply of output because its input seller's input seller was able to gather more resources in the reallocation process. Ultimately, it is the topology of the production network along with the distribution of quantity constraints across sectors that determines which sectors expand their supply of output beyond the steady-state level in response to the lockdown.

Remark 4 The supply of output of certain sectors increases during the lockdown period. This is because the heterogeneous quantity constraints redirect resources from some sectors to others. Put differently, some sectors have more access to resources than during normal times. Though no sector is permitted to produce more than its pre-lockdown output at any given time step, sectors can increase supply by carrying unsold inventory from previous time steps.

3.4 Results as the upper bound of effect of lockdowns

The numerical results presented in this paper are upper bounds of the impact of the lockdown on GDP. There are two reasons for why our calibration of the model will tend to overestimate the impact of the lockdown on GDP. The first of which has to do with the reallocation of demands across firms within the same sector. Note that since we calibrate the model at the level of sectors rather than firms, it is implicitly assumed that there is no non-neutral reallocation of demands across firms within the same sector, where by "non-neutral", we mean that no reallocation of demands such that the net-demand to some firms increases or decreases. Note that prima facie this would be a problematic assumption if the lockdowns were such that they affected some firms in a sector but not others. In which case, buyers would redistribute demand from firms under lockdown to firms that are not under lockdown. By and large, the lockdown in India that is being studied in this paper did not differentiate between firms within a sector. More specifically, the lockdowns were sector specific. Furthermore, for much of the period of the lockdowns studied in this paper the central government imposed a lockdown across the country, which meant that the location of a firm did not allow it to produce more than its peers from the same sector operating in another location. That said, India is a vast and varied country, with state capacity to enforce the lockdown varying across states and districts. In such a setting, it would be naive to assume that the lockdown was equally stringently enforced across all districts. Overall, there was probably some redistribution of demands across firms within the same sector. This redistribution of demands is likely to have dampened the impact of the lockdown on GDP. The exact size of the dampening will depend on the size of the redistribution of demands, for which we do not have data.

The second point reason for why our calibration of the model overestimate the impact of the lockdown on GDP has to do with the use of Cobb–Douglas rather than a CES production function. Note that CES production functions allow for substitutability between inputs based on prices. Put differently, with CES production functions the optimal share of expenditure on the inputs supplied by a particular firm depends on the prices charged by the firm, whereas with Cobb Douglas the share of expenditure does not depend on prices. CES therefore allow us to better model the changes in the inputs supplied by different firms in response to changes in prices. We, however, calibrated the model using Cobb Douglas production functions by assigning sectoral-flow weights as the Cobb–Douglas exponents. This seemed to be a natural way to calibrate the model, particularly in the absence of any data on how sectoral-flows change in response to sectoral prices. Since the model is calibrated at the level of sectors rather than firms, the redistribution of demand in question is the redistribution between inputs from different sectors rather than different firms. It would not be unreasonable to assume that in the short-run, inputs from different sectors are not good substitutes for each other. Furthermore, the real issue during the lockdowns was the passage of the shock through quantity bottlenecks rather than the ability or inability of firms to substitute inputs in response to rapid changes in prices. That said, in so far as our assumption of Cobb Douglas generates lower responsiveness to input prices than CES, our calibration will tend to overestimate

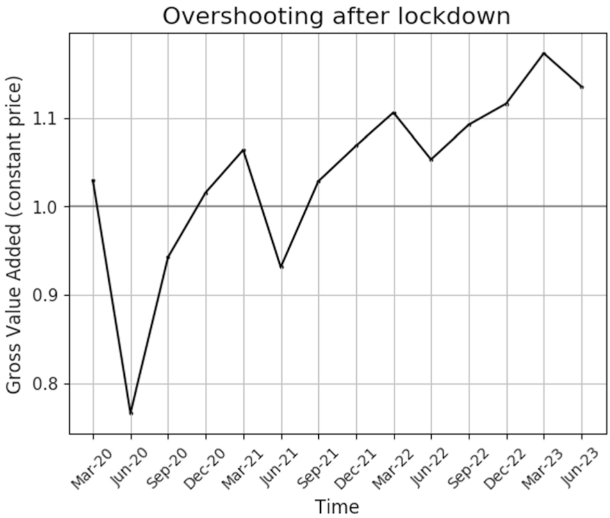


Fig. 5 Gross Value Added (at constant prices) normalized to the months in the year 2019

the impact of the lockdown. Unfortunately, we do not have an exact measure of this overestimation due to the paucity of firm level input use data for India.

3.5 Did the Indian economy experience an overshooting recovery?

Figure 5 presents gross value added (GVA) at constant prices from March 2020 to June 2023. The data are normalized to the year 2019.[11] The data shows somewhat of an interesting pattern. Economic output decreases in response to the lockdown, with the trough being June 2020. Output begins to recover after June 2020, by March 2021 output sizeably exceeds its levels in 2019 and 2020. There is a second dip in output after March 2021 because of the lockdowns that were imposed in response to the second wave of COVID. Output begins to recover after June 2021, once the second wave receded and the lockdown lifted. By March 2022, output was nearly 8% than in March 2020. It is difficult to decompose this increase in the level of output into 'regular economic growth' and 'the overshooting of GDP because of the lockdown'. On the one hand, it would not be reasonable to presume that the usual mechanisms of growth are working full-steam during the COVID-lockdown. While on the other hand, one may be going too far in presuming that the usual mechanisms of growth were dead during the lockdown. Ultimately, a decomposition of the excess output in March 2022 compared March 2020 depends on one's presumptions about the workings of the 'usual growth process' during the COVID years.

That said, given the fact that the Indian economy experienced two series of lockdowns and a pandemic between March 2020 and March 2022, it would perhaps be unwise to presume that usual growth process account for the full difference of about

[11] For example, the March 2020 value is the ratio of GVA in March 2020 to GVA in March 2019, the June 2021 value is the ratio of GVA in June 2021 to GVA in June 2019.

8% in output levels in the 2 year period. The pandemic was a period of considerable loss of lives, increased bankruptcy filings, financial crunch, input shortages, and labor shortage as migrant workers returned home. Such a scene resembles more an economy on its knees than an economy going through the ordinary process of growth via accumulation of factors of production and improvements in technology. There is therefore some evidence to suggest that there may have been some overshooting of GDP in the recovery from the COVID-19 lockdowns.

If we had granular data on firm buyer–seller relations between then nearly fifty million firms in the Indian economy, we could calibrate our model to such data and thereby generate out-of-sample predictions that may be able to more accurately match the time dynamics of the recovery of output after the lockdown. Calibrating the model to data on flows between a few hundred sectors means that much of the dynamics that our model is designed to study is omitted. Therefore, the overshooting result of our paper can only be compared in a 'qualitative sense' with the exact time series of output of the Indian economy.

4 The network multiple

We have so far examined the transient dynamics of sectoral supply of output without distinguishing between the portion of the changes in sectoral output that emerges from 'heterogeneity in the quantity constraints' from the portion that emerges from 'heterogeneity in their network positions'. Put differently, how much does the supply of output of a sector, during the transient period, differ from the quantity constraint imposed on the sector? One measure of such network effort is the ratio of the 'minimum supply of output of a sector' to its 'quantity constraint'. We call this ratio the 'network multiple'. Table 3 lists the network multiple for all the sectors in the input–output table in ascending order. A multiple of less than 1 means that the sector's output at its minimum is lower than the quantity constraint imposed by the lockdown. The table shows that 'rubber and plastics', 'transport equipment', and 'chemicals and chemical products' are the sectors with the lowest multiple. While 'Coke, refined petroleum, and nuclear fuel', 'Sale, maintenance, and repair of motor vehicles and motorcycles; retail sale of fuel', and 'Wholesale trade and commission trade, except of motor vehicles and motorcycles' have the highest multiple. Note that for several sectors the multiple is greater than 1, which means that their lowest supply of output during the transient was greater that the quantity constraint imposed. These sectors were able to build up inventory by drawing resources from sectors who faced more severe constraints because of their network positions.

Note that the network multiples presented in Table 3 must be interpreted with caution. Our way of computing the multiple assumes away a variety of factors that may considerably change the ranking of sectors in terms of their network effect. Not the least of which is the heterogeneity among sectors in their ability to carry inventory from one period to the next. In some sectors, inventory can be carried using simple storage technology. In other sectors, imperfections of storage technology will mean that some of the inventory will be lost at each time period. And finally, there are some sectors where there is no sense in which one can carry inventory, many

Table 3 Sectoral network multiple

Code	Sector	Network multiple
c10	Rubber and plastics	0.20
c15	Transport equipment	0.22
c9	Chemicals and chemical products	0.22
c16	Manufacturing, nec; recycling	0.23
c14	Electrical and optical equipment	0.23
c4	Textiles and textile products	0.24
c5	Leather, leather products, and footwear	0.24
c7	Pulp, paper, paper products, printing, and publishing	0.24
c13	Machinery, nec	0.26
c6	Wood and products of wood and cork	0.33
c11	Other nonmetallic minerals	0.33
c22	Hotels and restaurants	0.35
c33	Health and social work	0.40
c23	Inland transport	0.43
c18	Construction	0.45
c26	Other supporting and auxiliary transport activities; activities of travel agencies	0.46
c25	Air transport	0.47
c24	Water transport	0.51
c30	Renting of M &Eq and other business activities	0.68
c27	Post and telecommunications	0.71
c17	Electricity, gas, and water supply	0.85
c3	Food, beverages, and tobacco	0.88
c32	Education	0.93
c29	Real estate activities	0.95
c1	Agriculture, hunting, forestry, and fishing	0.97
c2	Mining and quarrying	1.16
c34	Other community, social, and personal services	1.18
c12	Basic metals and fabricated metal	1.28
c28	Financial intermediation	1.50
c21	Retail trade, except of motor vehicles and motorcycles; repair of household goods	1.55
c20	Wholesale trade and commission trade, except of motor vehicles and motorcycles	1.55
c19	Sale, maintenance, and repair of motor vehicles and motorcycles; retail sale of fuel	1.55
c8	Coke, refined petroleum, and nuclear fuel	1.65

services come within this category. A barber would be hard-pressed to carry inventory of haircuts forgone. All our simple computations show is that there is sizeable heterogeneity among sectors in how the production network affects their ability to function after the lockdown.

4.1 The relation between network multiple and the position of a sector in the supply chain

Presumably the position of a sector within the supply-chain can influence the extent to which the direct effects of the lockdown are amplified by the network of relations between firms. For instance, a sector which is fairly downstream supplying a basic consumer good may be somewhat less affected by the lockdown dynamics because it receives a relatively steady flow of consumer demand. In fact, such a sector may even temporarily benefit from the imposition of quantity constraints because it is in a position to outbid other sectors in the purchase of commonly used intermediate inputs. To study this issue, we develop a measure of the 'downstreamness' of sector and then relate this measure to the size of the network multiple computed in Sect. 4. We call the measure of 'downstream-ness' as the supply chain index ($\mathbf{v}$) defined as follows:

$$\mathbf{v} = \lambda \mathbf{A}^{\mathbf{T}} \mathbf{v} + \mathbf{u} \tag{14}$$

where $\mathbf{v}$ is the $m \times 1$ vector, $\mathbf{A}$ is the $m \times m$ adjacency matrix of flows between sectors with a_{ij} denoting the flow of money from i to j (or equivalently the flow of goods from j to i). $\mathbf{u}$ is a $m \times 1$ vector of weights of sectors with respect to the final consumer: u_i is the share of sector $i's$ output that goes to the final consumer. We normalize the weights $\mathbf{A}$ so that each sectors outgoing weights in terms of goods sum to 1, i.e., $\sum_i a_{ij} + u_i = 1$. $\lambda \in (0, 1)$ is a scalar. $\mathbf{v}$ is therefore given by:

$$\mathbf{v} = \left(\mathbf{I} - \lambda \mathbf{A}^{\mathbf{T}}\right)^{-1} \mathbf{u} \tag{15}$$

Finally, we define $\hat{\mathbf{v}}$ as $\mathbf{v}$ normalized by dividing each v_i by the maximum value in vector $\mathbf{v}$. According to our definition if $\hat{v}_i < \hat{v}_j$, then sector j is more downstream than sector i.

Remark 5 The supply chain index $\hat{\mathbf{v}}$ is a form of weighted Bonacich centrality where the weights are proportional to the sectoral expenditures of the household or final consumer.[12]

We computed the normalized supply chain index $\hat{\mathbf{v}}$ using data on sectoral linkages in the IO table with $\lambda = 0.9$. Table 4 orders the 33 sectors considered in this paper according to the supply chain index. 'Agriculture, hunting, forestry, and fishing' is the most downstream sector and ' Water transport' is the most upstream sector.

Figure 6 presents the relation between the supply chain index and the network multiple. There is a mild positive relation between the supply chain index and the

[12] The supply chain index $\hat{\mathbf{v}}$ is also related to measures of Total Forward Linkages developed by Antras et al. (2012) and Miller and Temurshoev (2017).

Table 4 Supply chain index

Code	Sector	SCI $\hat{v}$
c1	Agriculture, hunting, forestry, and fishing	0.254
c18	Construction	0.174
c23	Inland transport	0.160
c21	Retail trade, except of motor vehicles and motorcycles; repair of household goods	0.148
c3	Food, beverages, and tobacco	0.121
c29	Real estate activities	0.107
c20	Wholesale trade and commission trade, except of motor vehicles and motorcycles	0.091
c4	Textiles and textile products	0.086
c28	Financial intermediation	0.080
c15	Transport equipment	0.078
c16	Manufacturing, nec; recycling	0.076
c9	Chemicals and chemical products	0.072
c30	Renting of M &Eq and other business activities	0.071
c12	Basic metals and fabricated metal	0.069
c8	Coke, refined petroleum, and nuclear fuel	0.067
c31	Education	0.062
c22	Hotels and restaurants	0.060
c17	Electricity, gas, and water supply	0.046
c14	Electrical and optical equipment	0.043
c13	Machinery, nec	0.038
c32	Health and social work	0.037
c24	Other community, social, and personal services	0.036
c2	Mining and quarrying	0.030
c11	Other nonmetallic minerals	0.029
c10	Rubber and plastics	0.026
c7	Pulp, paper, paper products, printing, and publishing	0.017
c27	Post and telecommunications	0.016
c6	Wood and products of wood and cork	0.014
c19	Sale, maintenance, and repair of motor vehicles and motorcycles; retail sale of fuel	0.012
c5	Leather, leather products, and footwear	0.011
c26	Other supporting and auxiliary transport activities; activities of travel agencies	0.007
c25	Air transport	0.004
c24	Water transport	0.001

network multiple.[13] The positive relation emerges from the fact that within our setting, while firms face quantity constraints, the representative household does not face any explicit constraint on final demand. This means that compared to upstream

[13] The linear regression line plotted in Fig. 6 has a positive slope of 0.23 with a p value of 0.19 and r-value of 0.23. There is little reason to presume a linear relation between the two variables, both of which involve nonlinear transformations of the network of relations between firms. A linear regression is merely a starting point to examine such complex relations.

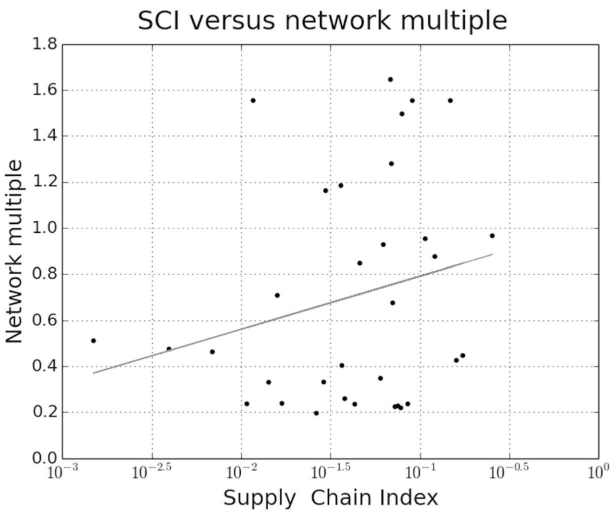


Fig. 6 Relation between the Supply Chain Index and the Network Multiple

firms, downstream firms face less interruption in demand. This demand side effect diverts resources away from upstream firms and toward downstream firms. Put differently, downstream firms benefit from their network position relative to upstream firms in the transient dynamics that emerge from the imposition of the lockdown. And this resource reallocation across the supply chain generates the mild positive relation marked in Fig. 6.

5 Price stickiness and overshooting recovery

We had so far assumed that prices are fully flexible after the lockdown is lifted. The purpose of this assumption was to illustrate transient network dynamics within a relatively simple setting. Empirical evidence, however, suggests that prices are sticky. In this section we introduce price-stickiness to study our model dynamics under a more realistic setting. Each firm sets its price as a linear combination of the price charged the previous time step and the current market clearing price. More specifically, the price of firm i at time step t is given by the following equation:

$$p_i^t = \rho p_i^{t-1} + (1-\rho)p_i^{*t} \tag{16}$$

where $\rho \in [0, 1]$ is a parameter measuring the speed of price adjustment and p_i^{*t} is the local market clearing price of firm i. Goods are allocated as given by the following equations[14]:

[14] Non-market-clearing prices generate nonzero excess demands. In case of positive excess demand, goods are rationed in proportion to the nominal demand from different buyers. In case of negative excess demand, firms carry inventory over to the next time step. The inventory so carried is treated no differently from the output produced at the next time step. In other words, the inventory is added to the output produced to determine the price.

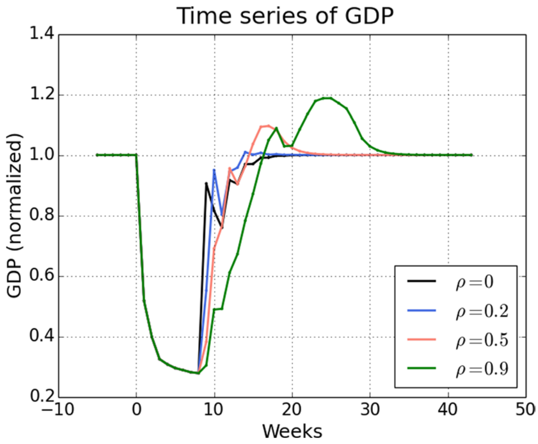


Fig. 7 Time series of GDP after the Lockdown for different levels of price-stickiness

- If the price of a good p_j^t is greater than or equal to the local market clearing price p_j^{*t} then for all $i, j \in N$:

$$y_{ij}^t = \frac{a_{ij} w_i^t}{p_j^t} \text{ (inputs allocation)} \tag{17}$$

$$x_i^t = \frac{\beta_i w_0^t}{p_i^t} \text{ (consumption allocation)} \tag{18}$$

- Otherwise, if the price of a good p_j^t is less than the market clearing price p_j^{*t}, agents are rationed proportionally to their demand and one has for all $i, j \in N$:

$$y_{ij}^t = r_i^t a_{ij} w_i^t \quad \text{(inputs allocation)} \tag{19}$$

$$x_i^t = r_0^t \beta_i w_0^t \quad \text{(consumption allocation)} \tag{20}$$

where $r_i^t = \frac{a_{ji} w_j^t}{\sum_{j \in N} a_{ji} w_j^t + \beta_i w_0^t}$ and $r_0^t = \frac{\beta_i w_0^t}{\sum_{j \in N} a_{ji} w_j^t + \beta_i w_0^t}$.

The introduction of price stickiness within our model generates what we call 'overshooting recovery'. Put differently, the GDP increases above its steady-state level after the relaxation of the lockdown before returning to equilibrium. Figure 7 shows the time series of GDP with different values of the price-stickiness parameter ρ. Note that higher levels of price-stickiness ρ are associated with greater size of the overshooting of GDP. Figure 8 plots the size of the overshooting of GDP for different values of price stickiness. The size of overshooting is measured as the sum total of GDP above its steady-state level during the transient to the new equilibrium. The

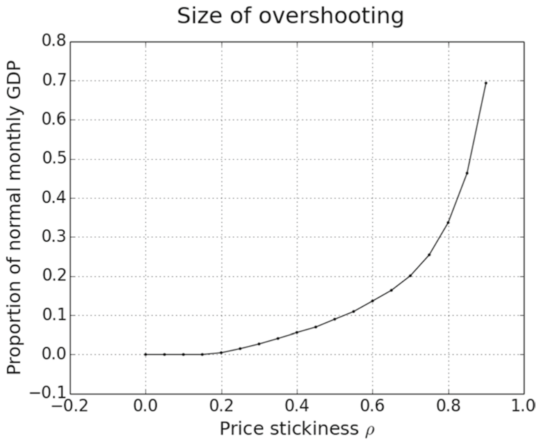


Fig. 8 The relation between the size of overshooting of GDP and price-stickiness

y-axis of Fig. 8 marks the 'sum of overshooting' normalized by monthly GDP. Figure 8 shows that the size of overshooting increases with price-stickiness ρ.

The overshooting of GDP after the removal of the lockdown is a consequence of the accumulation of inventory in transient phase *after* the lifting of the lockdown rather than the accumulation of inventory during the lockdown. But before we get into the details of inventory accumulation after the lifting of the lockdown, it is important to shed light on the nature of inventory accumulation during the lockdown. While there is some accumulation of inventory during the lockdown period, it is not as if sectors continue producing at pre-lockdown levels and store the unsold output as inventory. Most sectors experience considerable reductions in production levels because of the non-availability of inputs. The accumulation of inventory by any sector causes those who use the good as an input to contract production due to lack of inputs. Therefore inventory accumulation in some sectors causes reductions in output in other sectors. Furthermore, in some sectors the inventory accumulation occurs despite sizeable reductions in production levels. This is because the demand for the output of these sectors declines more than the supply of inputs to these sectors. Overall, inventory build-up cannot go on unabated within our model simply because the accumulation of inventory in one sector generates a general reduction in output across the economy. This is because the accumulation of inventory means that those goods do not flow as inputs into the production process.

The overshooting of GDP is not because inventories that are accumulated during the lockdown are released in the market after the lockdown. If that were the case, overshooting would occur with perfectly flexible prices, but there is no overshooting of GDP with flexible prices (see black line in Fig. 6). It turns out that the inventory accumulated during the lockdown is not sufficient to cause an overshooting of GDP. The overshooting of GDP happens for a very different reason. During the early stages of recovery *after* the end of the lockdown, there are considerable changes in local demand and supply in specific markets. The mutual adjustments of different sectors in response to the new circumstances causes positive and negative excess demands in different markets at different time steps. When prices are sticky, negative excess demand generates an accumulation of inventory. The inventory is released in the latter stages of recovery as prices approach general equilibrium prices and local demand-supply changes dampen. This injection of inventory cause an overshooting of GDP. The size of the injection depends on the size of the inventory built-up during the early stages of recovery *after* the lockdown. And the size of inventory built-up depends on the stickiness of prices ρ, which is why higher stickiness of prices causes a greater overshooting of GDP.

Remark 6 The economy experiences a recovery with an overshooting for GDP when prices are sufficiently sticky. The overshooting of GDP occurs because the inventory accumulated in the early part of the transient post-lockdown period is injected into the economy in latter part of the period.

5.1 Price stickiness and the spillover of disequilibrium

Note that the relation between the degree of price-stickiness ρ and the size of overshooting is nonlinear: the size of overshooting increases at an increasing rate with price-stickiness. This phenomenon too has to do with the network of relations between firms. When one firm charges non-market clearing price at a certain time step, it generates a disturbance which reverberates through the buyer–seller relations. In the presence of price-stickiness, this disturbance can generate excess demands in various parts of the economic network. Put differently, an increase in price-stickiness and the concomitant charging of non-market clearing prices, causes the carrying of inventory not only by the firm affected by temporary mismatch between local demand and supply, but also by firms distantly related to it; though these transmissions take time and tend to decay with distance from the firm which originally faced the mismatch between its demand and supply. The production network therefore amplifies the impact of price stickiness on the transient time dynamics of GDP.

Ultimately, the super-linear of the overshooting of GDP has to do with the spilling over of disequilibrium from one market to another. The simplest setting in which we can understand the spilling over of disequilibrium is one in which there is no price stickiness, and therefore all markets at zero excess demand at all time steps. In such a setting, consider a shocks that temporarily disturbs equilibrium in one market. For ease of analysis, consider a monetary shock that takes the form an injection of money into one firm or one sector of the economy. The firm which receives new money will increase its demand for inputs. This in turn will increase the prices of those goods relative to other goods in the economy. The producers of these goods will experience an increase in the inflow of money, and will in turn increase the demand for goods they use as inputs. The users of the goods whose prices have risen will have to decrease production, which will generate an increase in the prices of their output. There will also be places in the economy where prices will temporarily decrease simply because local demand contracts relative to local supply. The simplest of these price decreases will be of the good that is produced by the firm that receives the initial money injection, that firm was able to increase output at the first time step while the firms that demand this output are in no position to increase demand, leading to a decrease in the price of the good. Overall, in the short-run, the deviation of relative price in one market generates deviations in relative prices in other markets. Disequilibrium spills over from one market to another. In the long-run, this spillover process (or 'cumulative process' to use a Wicksellian phrase) takes the economy to a new equilibrium in which relative prices are the same as in the pre-shock equilibrium. We have studied the spillover process that ensues from a monetary shock in Mandel et al. (2019) and Mandel and Veetil (2021).

The afore noted spillover takes place through price-adjustments. Similar spillover can take place through quantity-adjustments. Consider a scenario in which *Sector A* decreases the demand for inputs from *Sector B* in response to a lockdown. If prices are not fully flexible, the decrease in demand for input will generate some increase in allocation of the good to sectors than *Sector A* and an increase in the build-up of inventory in *Sector B*. Put differently, some portion of the output not sold to *Sector A* is given to other sectors and the remaining portion is stored as inventory. Suppose *Sector C* and *Sector D* are the two sectors that receive additional input from *Sector B*. In response to the additional availability of input, *Sector C* and *Sector D* will increase production. Suppose the prices charged by *Sector C* and *Sector D* are not fully flexible. Much like *Sector B*, *Sector C* and *Sector D* will meet the increase in output relative to demand partly by increasing allocation to other sectors and partly by building inventory. And 'the increase in allocation to other sectors' is capable of generating inventory build-up in those sectors. Inventory build-up, and thereby disequilibrium, spills from one market to another. Note that the magnitude of inventory build-up and its spillover depends not only on price-stickiness in individual markets but also on how these markets relate to each other. More specifically, a high stickiness of price in one market can amplify the build-up of inventory that originates from a market to which it is directly connected. For instance, the stickiness of price of *Sector B* determines how much additional input is sold to *Sector C*, which in turn determines the expansion in output of *Sector C*. As to what portion of this increase in output is kept as inventory by *Sector C* depends on the stickiness of price of *Sector C*. Therefore, ultimately the build-up of inventory by *Sector C* depends not only on the stickiness of its own price but also on the stickiness of price of *Sector B*.

Another way to follow the spillover of disequilibrium within our model is to follow the changes in the flow of money across sectors. Within our model, changes in demand and supply in specific markets depends on changes in the flow of money between sectors. Sectors that experience an increase in money holdings increase demand one time-step and expand supply next time-step. Sectors that experience a decrease in money holdings decrease demand one time-step and contract supply next time-step. Inventory build-up occurs when the shifts in demand and supply in response to changes in money flows are such that demand contracts relative to supply. Ultimately, price stickiness in one market influence inventory build-up in another because changes in flows of money are influenced by the network structure of the stickiness of prices.[15]

[15] No one has so far studied the influence of the stickiness of prices in a multi-market setting, wherein price stickiness in one market can amplify the effect of price stickiness in another market. The super-linearity of the overshooting of GDP suggests that price stickiness interacts across markets related to each

Remark 1 The overshooting of GDP increases super-linearly with an increase in price stickiness. This super-linearity originates from the interaction of price stickiness with the network structure of production.

6 Concluding remarks

The problem of curbing economic activity to reduce the spread of a pathogen is in essence an optimization problem. More specifically, given the fact that the pathogen can spread via human contact in the course of economic activity, the problem is one of minimizing the cost of economic restrictions for any desired rate of the spread of the pathogen. This large nonlinear optimization problem is unlikely to lend itself to analytical representation, let alone analytical solutions. A more natural approach to modeling such a problem is the agent-based setting, which involves building synthetic societies in silico and then simulating their behavior under various parametric conditions. Such a model would need to dock an epidemiological model of disease transmission with a model of economic activity. In this paper, we have presented some of the essential elements of one such model of economic activity. We have argued that the network of relations between economic agents are an important determinant of the cost of imposing economic restrictions. On the one hand, quantity constraints can percolate through the economic network thereby increasing the cost of lockdowns well beyond the simple sum of the restriction on each sector. On the other hand, the reallocation of resources across sectors in response to quantity constraints can increase production in some sectors thereby decreasing the overall cost of economic restrictions. The topology of the economic network and the sectoral distribution of economic restrictions are therefore among the primary determinants of the cost of curbing economic activity to limit the spread of pathogens.

While agent-based models are capable of granular calibration to real world data, we have had to calibrate our model to India's input–output table with three dozen odd sectors. Presumably there are on the order of 10^6 firms in India. But data on the buyer–seller relations of these firms are not readily available. Nor are the data on their geographic locations and product mix, all of which would be necessary to calibrate the model at the firm level to study how the flows of inputs are affected by pandemic policy. Note that data on sectoral flows are no substitute for firm level data. Sectoral data aggregates and thereby smoothers sizeable flows between firms.

Footnote 15 (continued)

other as suppliers of intermediate inputs. Our impression is that macroeconomic dynamics is influenced by the 'network structure of price stickiness' by which we mean the distribution of price stickiness across markets related to each other via their input–output relations. Consider two economies, $\mathcal{E}_1$ and $\mathcal{E}_2$, each with n sectors. Assume that the network of buyer–seller relations between sectors in two economies is given by adjacency matrices M_1 and M_2. Suppose further that the distribution of price stickiness in both economies is given by ϕ. It may well be that the time dynamics of aggregate variables in response to fiscal and monetary shocks differ in two economies because $\mathcal{E}_1$'s time dynamics is driven by the relation between M_1 and ϕ, whereas $\mathcal{E}_2$'s time dynamics is driven by the relation between M_2 and ϕ. Most workhorse macroeconomic models with price stickiness implicitly assume that the way in which ϕ is embedded on M_1 and M_2 does not matter in the propagation of fiscal and monetary shocks. This seems to be far too heroic an assumption.

And in so far as firm level flows drive production activity, sectoral aggregation can significantly under or overestimate the cost of economic restrictions. The cost will be underestimated to the extent that firms within a sector buy inputs from each other and therefore are affected by the restrictions on their input providers. The cost will be overestimated to the extent that there may be reallocation of resources within a sector in response to economic restrictions.

Data on firm level flows are present in the digital vaults of the tax department simply because of value added tax filing requisite information on input sellers. From a public policy point of view, it would be sensible to use such data to calibrate a large agent-based model which couples epidemiological and economic dynamics. Such a model can then serve as a testbed for computationally experimenting with various policy alternatives. One could, for instance, study the consequences of different distributions of geographic and sectoral restrictions on economic activity. Some distributions may well prove to be superior to others in minimizing economic costs for target levels of the spread of a pathogen. Granularly calibrated large agent-based models can therefore serve as guides to public policy decisions. The development of such public policy capabilities will ultimately depend on institutions that enable productive interactions between people capable of bringing data and models together.

Declarations

Conflict of interest This paper did not receive any source of funding. There is no financial or non-financial conflict of interest. This paper complies with the ethical standards of JEIC.

References

Acemoglu D, Carvalho VM, Ozdaglar A, Tahbaz-Salehi A (2012) The network origins of aggregate fluctuations. Econometrica 80(5):1977–2016
ADB (2020) India input–output table 2017. Asian Development Bank
Antras P, Chor D, Fally T, Hillberry R (2012) Measuring the upstreamness of production and trade flows. Am Econ Rev 102(3):412–416
Arthur WB (2006) Out-of-equilibrium economics and agent-based modeling. Handb Comput Econ 2:1551–1564
Axtell R (2000) Why agents?: On the varied motivations for agent computing in the social sciences. In: The Brookings Institution working paper 17. Center on Social and Economic Dynamics
Baqaee DR, Farhi E (2020)Supply and demand in disaggregated Keynesian economies with an application to the Covid-19 crisis. In: Discussion paper DP14743. Centre for Economic Policy Research
Barrot JN, Grassi B, Sauvagnat J (2020) Sectoral effects of social distancing. COVID Econ 3:85–102
Bonet-Morón J, Ricciulli-Marín D, Pérez-Valbuena GJ, Galvis-Aponte LA, Haddad EA, Araújo IF, Perobelli FS (2020) Regional economic impact of COVID-19 in Colombia: an input–output approach. Reg Sci Policy Pract 12:1123–1150
Borrill PL, Tesfatsion L (2011) Agent-based modeling: the right mathematics for the social sciences?. In: Davis JB, Wade Hands D (eds) The Elgar companion to recent economic methodology, chapter 11. Edward Elgar Publishing, pp 228–258
Dev M, Sengupta R (2020) Covid-19: impact on the Indian economy. In: Indira Gandhi Institute of development research working paper no. 2020-013

Epstein JM (1999) Agent-based computational models and generative social science. Complexity 4(5):41–60
Fadinger H, Schymik J (2020) The effects of working from home on covid-19 infections and production: a macroeconomic analysis for Germany. In: Working paper
Giammetti R, Papi L, Teobaldelli D, Ticchi D (2020) The Italian value chain in the pandemic: the input–output impact of Covid-19 lockdown. J Ind Bus Econ 47:483–497
Goyal A (2020) Post Covid-19: recovering and sustaining India's growth. Indian Econ Rev 55:161–181
Gualdi S, Mandel A (2016) On the emergence of scale-free production networks. J Econ Dyn Control 73:61–77
Gualdi S, Mandel A (2018) Endogenous growth in production networks. J Evol Econ 29(1):1–27
IFO-Institute (2020) The economic costs of the coronavirus shutdown for selected European countries: a scenario calculation
Inoue H, Todo Y (2020) The propagation of the economic impact through supply chains: the case of a mega-city lockdown against the spread of COVID-19. Covid Econ 2:43–59
Inoue H, Murase Y, Todo Y (2021) 'Do economic effects of the anti-COVID-19 lockdowns in different regions interact through supply chains?. In: Working paper
Kanitkar T (2020) The COVID-19 lockdown in India: impacts on the economy and the power sector. Glob Transit 2:150–156
Livan G, Novaes M, Perpaolo V (2017) Introduction to random matrices, theory and practice. Springer, Berlin
Mahajan K, Tomar S (2021) COVID-19 and supply chain disruption: evidence from food markets in India. Am J Agric Econ 103(1):35–52
Malinvaud E (1977) The theory of unemployment reconsidered. Blackwell, Oxford
Malinvaud E (1981) Econometrics faced with the needs of macroeconomic policy. Econom J Econom Soc 49:1363–1375
Malinvaud E (1982) An econometric model for macro-disequilibrium analysis. Springer, Berlin, pp 239–256
Mamgain RP (2021) Understanding labour market disruptions and job losses amidst COVID-19. J Soc Econ Dev 23:301–319
Mandel A, Veetil VP (2020a) Disequilibrium propagation of quantity constraints: application to the COVID lockdowns. In: SSRN working paper no
Mandel A, Veetil VP (2020b) The economic cost of COVID lockdowns: an out-of-equilibrium analysis. Econ Disaster Clim Change Extreme Events 4:431–451
Mandel A, Veetil V (2021) Monetary dynamics in a network economy. J Econ Dyn Control 125:104
Mandel A, Taghawi-Nejad D, Veetil VP (2019) The price effects of monetary shocks in a network economy. J Econ Behav Organ 164:300–316
McCann F, Myers S (2020) COVID-19 and the transmission of shocks through domestic supply chains. In: Working paper
Miller RE, Temurshoev U (2017) Output upstreamness and input downstreamness of industries/countries in world production. Int Reg Sci Rev 40(5):443–475
Osotimehin S, Popov L (2020) Sectoral impact of COVID-19: cascading risks. In: Opportunity and inclusive growth institute working paper no. 30
Richiardi M, Bronka P, Collado D (2020) The economic consequences of COVID-19 lock-down in the UK: an input–output analysis using consensus scenarios. In: Working paper
Sahoo P, Ashwani (2020) COVID-19 and Indian economy: impact on growth, manufacturing, trade and MSME sector. Glob Bus Rev 21(5):1159–1183
Sengupta S (2020) Coronavirus, population and the economy: a long-term perspective. Indian Econ J 68(3):323–340
Taylor L (1983) Structuralist macroeconomics. Basic Books, New York
Vidya CT, Prabheesh KP (2020) Implications of COVID-19 pandemic on the global trade networks. Emerg Mark Finance Trade 56(10):2408–2421