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Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components

In Milton Friedman's model, output cannot exceed a ceiling level but occasionally is "plucked" downward by recession, implying fluctuations are asymmetric, recessions transitory, and recessions duration dependent though expansions are not. The empirical literature lends support, but formal modeling has been absent. The econometric model presented here encompasses both plucking and symmetric fluctuations around a stochastic trend. We find GDP is well characterized by the plucking model, implied recessions correspond to NBER reference cycles, and no role for symmetric cycles. Decomposition of the unemployment rate reveals a corresponding asymmetry and timing. Paths of ceiling output and trend unemployment are presented.

IN A REVIEW OF BUSINESS CYCLE STUDIES at the National Bureau of Economic Research, Milton Friedman (1964) noted a striking asymmetry in the correlations between succeeding phases of the business cycle: the amplitude of a contraction is strongly correlated with the succeeding expansion although the amplitude of an expansion is uncorrelated with the amplitude of the succeeding contraction. This lead him to propose the "plucking" model of business cycles, likening the path of output to a string attached to the underside of a board which is plucked downward at irregular intervals. The board represents an upper limit on output set by resources and the way they are organized. Though the extent of the decline will vary across episodes, output will always rebound to the ceiling level. Subsequent literature contains much evidence of the kind of asymmetry Friedman described, but not a formal model capable of estimating the importance of downward shocks and testing the plucking hypotheses against a symmetric trend-plus-cycle alternative. That is the objective of this paper.

That the plucking model has been controversial (Friedman 1993) is surprising since

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it is implicit in the practice of measuring the "deflationary gap" as the amount by which actual output falls below a trending ceiling that is achieved only at peaks which has been widely accepted since Okun (1962); see also Delong and Summers (1988). It is also implicit in what the profession has taught undergraduates since Samuelson (1948), namely, that the economy operates on or within a production possibilities frontier. Recession is depicted as a point inside the frontier where resources are idle. Unless recessions are permanent, this implies that transitory fluctuations in output are asymmetric in the negative direction.

Another kind of business cycle asymmetry has been recognized at least since Keynes (1936) who noted that "the substitution of a downward for an upward tendency often takes place suddenly and violently, whereas there is, as a rule, no such sharp turning point when a upward is substituted for a downward tendency." While the plucking model certainly implies that downturns are steeper (peak to trough) than are expansions (trough to peak) as Keynes had observed, that kind of asymmetry is also implied by a model in which recessions are due to infrequent but large negative permanent shocks. What distinguishes the plucking model for a purely real model of business fluctuations is the prediction that negative shocks are largely transitory, while positive shocks are largely permanent. Thus, recessions are like the common cold: they come on suddenly and recovery follows a fairly predictable course, but the time that has passed since the last cold is of no use in predicting when the next will occur, or its severity.

Section 1 of the paper reviews briefly a growing body of literature on business cycle asymmetry that supports the plucking model. Section 2 introduces a formal econometric model which encompasses permanent shocks, symmetric fluctuations, and asymmetric plucks as competing explanations for recessions. Markov switching moves the economy between the normal state and recession where large asymmetric but transitory shocks are allowed to occur if the data indicate. Empirical results for U.S. real GDP and unemployment are presented in section 3, and these give strong support for the plucking model. Estimates of the permanent and transitory components of U.S. real GDP and unemployment rate provide a picture of the postwar U.S. economy operating much of the time near, but never substantially above, a ceiling level but occasionally plucked below that level by shocks occurring at roughly the NBER business cycle peaks but which dissipate quite rapidly, though less rapidly for unemployment. Section 4 draws some conclusions for future business cycle research.

1. ASYMMETRY IN THE BUSINESS CYCLE LITERATURE

In an influential paper which initiated the modern literature on business cycle asymmetry, Neftci (1984) presented formal statistical evidence of the kind of asymmetry that was informally observed by Keynes: the behavior of the unemployment rate is characterized by sudden jumps and slower declines. Further evidence that recessions are steeper than expansions may be found in Delong and Summers (1986), Falk (1986), and Sichel (1993). While this type of asymmetry is certainly consistent with the plucking model, it is also consistent with models in which recessions are due to occasional permanent negative shocks as in the Markov-switching models of Hamilton (1989) and Lam (1990). However, Sichel (1993) also finds that recessions are deeper than expansions are tall, an asymmetry that is implied by models in which recessions are transitory but not by models in which recessions are due to permanent shocks.

Confirming the relationship between the depth of a recession and the strength of the subsequent recovery that motivated Friedman, Sichel (1994) shows that postwar real GDP exhibits "peak-reverting behavior." A variable measuring the depth of a recession, how far output has fallen below its prior peak, contains information useful for predicting the subsequent growth rate of real GDP. He argues that this finding suggests the existence of a third, high-growth recovery phase, in addition to the usual recession and expansion phases of the business cycle. This third phase is a feature of the model presented in this paper.

If recessions are primarily due to occasional transitory shocks while expansions primarily reflect permanent shocks as well as long-term growth, then we would expect to find that recessions exhibit duration dependence while expansions do not. That is, a recession, once it begins, will dissipate in a fairly predictable period of time, but the age of an expansion is not helpful in predicting the next recession. Diebold and Rudebusch (1990), Diebold, Rudebusch, and Sichel (1993), and Durland and McCurdy (1994) discuss business cycle duration dependence within univariate contexts, while Kim and Nelson (1998a) study the issue in a multivariate context. All find that postwar recessions are characterized by positive duration dependence; the longer a recession persists, the more likely it is to end. But duration dependence is not found for postwar expansions.

The literature on the persistence of shocks has generally imposed symmetry on measures of persistence; for example, Nelson and Plosser (1982), Campbell and Mankiw (1987), Watson (1986), and Cochrane (1988). When Beaudry and Koop (1993) allowed the impulse response of real GNP to be asymmetric, they found that negative innovations to output are much less persistent than positive ones. During recessions, output fluctuations are mostly transitory, while during normal times, output fluctuations are mostly permanent. They argue that macroeconomic theories that explain temporary changes in output may be relevant for understanding recessions and recoveries, while other macroeconomic theories which explain permanent changes in output may be more relevant for expansions. Indeed, Cover (1992) reports evidence of asymmetry in the effects of positive and negative money-supply shocks; negative monetary shocks have a larger and more important effect than do positive shocks, as suggested by Friedman (1964) in his discussion of the implications of the plucking model.

Finally, an important feature of Friedman's model is that there exists an upper limit to output or the maximum feasible output set by the available resources and methods of organizing them. Goodwin and Sweeney (1993) provide empirical evidence, based on a frontier production function, that such upper limits to output exist for a majority of eight OECD countries.

In spite of this copious evidence of business cycle asymmetry, there exists no formal econometric model of the business cycle that enables us to decompose measures of economic activity into a trend component and deviations from the trend that exhibit the kinds of asymmetry implied by the business cycle literature. Linear time series models such as ARIMA models or Clark's (1987) unobserved components model cannot account for asymmetry. The Markov-switching models of Hamilton (1989) and Lam (1990) allow for asymmetric behavior only in the growth rate or stochastic trend component of real output. Threshold models of the type investigated by Beaudry and Koop (1993) do not lend themselves as readily to testing and decomposition.

In the next section, we present and estimate a nonlinear time series model that incorporates asymmetric movements of output or unemployment away from trend and asymmetric persistence of shocks during recessions and expansions. Clark's (1987) linear unobserved components model and Friedman's plucking model are special cases.

2. AN ECONOMETRIC SPECIFICATION OF THE PLUCKING MODEL

Consider the following unobserved components model of economic fluctuations in which the log of real GDP (y_t) is decomposed into a trend component (τ_t) and a transitory component (c_t):

$$y_t = \tau_t + c_t . \tag{1}$$

To allow for regime shifts or asymmetric deviations of real GDP from its trend component, we assume that shocks to the transitory component are a mixture of two different types of shocks:

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + u_t^*, \tag{2}$$

$$u_t^* = \pi_{s_t} + u_t, \tag{3}$$

$$\pi_{s_t} = \pi S_t, \quad \pi \neq 0, \tag{4}$$

$$u_t \sim N\left(0, \sigma^2_{u,s_t}\right), \tag{5}$$

$$\sigma^2_{u,s_t} = \sigma^2_{u,0}(1 - S_t) + \sigma^2_{u,1} S_t, \tag{6}$$

$$S_t = 0, \ or \ 1, \tag{7}$$

where π_{s_t} is an asymmetric, discrete, shock which is dependent upon an unobserved variable S_t and u_t is the usual symmetric shock. S_t is an indicator variable that determines the nature of the shocks to the economy. During normal times, $S_t = 0$ (and thus, $\pi_{s_t} = 0$), and the economy is near the potential or trend output. During the recession times, $S_t = 1$, and the economy is hit by a transitory shock potentially with a negative expected value ($\pi_{s_t} = \pi < 0$):[1] Aggregate demand or other disturbances are plucking

1. In estimating the model, no constraint was given for the sign of π.

the output down. Equations (5)–(6) allow for the possibility that the variance of the symmetric shock u_t is different during the normal and recession times. To account for a persistence of normal periods or recession periods, we assume that S_t evolves according to a first-order Markov-switching process as in Hamilton (1989):

$$Pr[S_t = 1 \mid S_{t-1} = 1] = p, \tag{8}$$

$$Pr[S_t = 0 \mid S_{t-1} = 0] = q. \tag{9}$$

The above specification for the transitory component of output shares the same idea as in the literature on "stochastic frontier production function," originally motivated by Aigner, Lovell, and Schmidt (1977). They assume that the production process is subject to two economically distinguishable random disturbances, with different characteristics. Thus, they specify the distribution of disturbances to the production function as a mixture of a symmetric Normal distribution and a half Normal distribution truncated above at zero.[2] The nonpositive disturbance from a half Normal distribution reflects the fact that firm's output must lie on or below its production frontier. In our context, the recession periods with downward plucks are similar to periods when a firm uses factors suboptimally and is below its production frontier.

Turning to the specification of the stochastic trend component or the trend ceiling component, Friedman (1993) suggests that the potential output, or "the ceiling maximum feasible output," "may be approximated by a pure random walk, with all sorts of disturbances producing perturbations in it, including the recently popular technological disturbances."

$$\tau_t = g_{t-1} + \tau_{t-1} + v_t , \tag{10}$$

$$g_t = g_{t-1} + w_t , \tag{11}$$

$$w_t \sim N(0, \sigma_w^2), \tag{12}$$

$$v_t \sim N\left(0, \sigma_{v,s_t}^2\right), \tag{13}$$

$$\sigma_{vs_t}^2 = \sigma_{v,0}^2 (1 - S_t) + \sigma_{v,1}^2 S_t , \tag{14}$$

where the stochastic trend component, τ_t, in (10) is subject to different shocks: shocks to the level, v_t, and shocks to the growth rate, w_t. By modeling the trend growth term, g_t, in (11) to be stochastic, we allow for a possibility that the postwar GDP has undergone a productivity slowdown.[3] In (13)–(14), we also allow for the possibility that the variance of the shock to the level, u_t, may be different during the normal and recession times. Variance of the shock to the growth, w_t, on the contrary, is not likely to be systematically different during the normal and recession times.

2. Goodwin and Sweeney (1993) apply this specification to test for asymmetry of business cycle as implied by Friedman's "plucking" model.

3. As mentioned by Clark (1987), it does not seem to be appropriate to assume a constant growth in advance. Strictly speaking, this implies that GDP is I(2) rather than I(1), but in practice the estimated variance of the growth rate process is small.

Most of existing literature on the decomposition of real GDP based on linear unobserved components models (Clark 1987; Kuttner 1994; and Watson 1986, for examples) views economic fluctuations as symmetric movements around a stochastic trend. These models may be viewed as restricted versions of the model presented in this paper, the restrictions being $\pi = 0$, $\sigma^2_{v0} = \sigma^2_{v1}$, and $\sigma^2_{u0} = \sigma^2_{u1}$. Recent attempts to incorporate asymmetry in the time series models of economic fluctuations have been limited to the growth rate of real output itself or to the trend component. Hamilton's (1989) Markov-switching model of the growth of real GNP and Lam's (1990) generalized Hamilton model with a Markov-switching trend growth component and a symmetric transitory component are the two examples. Contrary to these models, the model presented in this section focuses on the asymmetric behavior of the cyclical or transitory component away from the stochastic trend or the trend ceiling component.

3. EMPIRICAL RESULTS

We estimate the model presented in section 2, using as data the log of quarterly real GDP and quarterly unemployment rate for the United States [1951:1–1995:3]. Recently, the Markov-switching model of Hammilton (1989) has been extended to a general state-space model by Kim (1993a, 1993b, 1994).[4] Once the model in section 3 is put into a state-space form, it can readily be estimated using the approximate MLE of Kim. For details of Kim's approximate MLE method, readers are referred to the Appendix.

3.1 Dynamics of Real GDP

Estimation results for the log of quarterly real GDP are summarized in Table 1. Model 1 (column 2) in Table 1 is an unrestricted version of our model with an asymmetric transitory component and model 4 (column 5) is an unrestricted version of Clark's (1987) linear unobserved components model with a symmetric transitory component. One notable difference between the estimates of the two models is the sum of the AR coefficients, $\phi_1 + \phi_2$, for the transitory component. It is 0.7969 for model 1 and 0.9458 for model 4, suggesting that the persistence of the transitory component gets markedly lower once asymmetry is accounted for. This is consistent with Perron (1990), who suggests that the standard unit root tests are biased toward nonrejection of the null of a unit root when the data generating process is stationary with a switching mean.

Another notable difference between the two models is the significance of the σ_w parameter, or the variance of the shock to the trend growth component. Comparing the log likelihood values for models 4 and 5, the LR test statistic for the hypothesis $\sigma_w = 0$ is 1.38 for the linear unobserved components model, failing to reject the null hypothesis. However, once asymmetry in the transitory component is accounted for in

4. A state-space representation is a very flexible form, and Kim's approach based on the approximate MLE allows a broad class of models to be estimated that could not be handled before.

TABLE 1

MAXIMUM LIKELIHOOD ESTIMATION OF THE MODEL: GDP

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5
σ_{v0}	0.0057	0.0056	0.0061	0.0056	0.0060
	(0.0010)[a]	(0.0013)	(0.0006)	(0.0013)	(0.0008)
σ_{v1}	0.0098	0.0102	0.0098	—	—
	(0.0018)	(0.0020)	(0.0017)		
σ_w	0.0007	—	0.0007	0.0002	—
	(0.0005)		(0.0005)	(0.0002)	
ϕ_1	1.2565	1.3344	1.2545	1.5346	1.5842
	(0.1260)	(0.1225)	(0.1219)	(0.1501)	(0.0345)
ϕ_2	−0.4595	−0.4585	−0.4733	−0.5888	−0.6274
	(0.1182)	(0.1027)	(0.1213)	(0.1155)	(0.0270)
π	−0.0111	−0.0102	−0.0108	—	—
	(0.0031)	(0.0026)	(0.0030)		
σ_{u0}	0.0024	0.0035	—	0.0061	0.0058
	(0.0020)	(0.0023)		(0.0013)	(0.0009)
σ_{u1}	0.0000	0.0000	—	—	—
	—[b]	—[b]			
p	0.7116	0.7353	0.7068	—	—
	(0.1157)	(0.1327)	(0.1118)		
q	0.9326	0.9331	0.9243	—	—
	(0.0336)	(0.0372)	(0.0350)		
Log Likelihood	585.71	583.99	585.46	578.52	577.83

[a]Standard errors of the parameter estimates are reported in the parentheses.
[b]ML estimates of σ_{u1} fell on the boundary ($\sigma_{u1} = 0$), which violates the regularity condition. To calculate standard errors we imposed $\sigma_{u1} = 0$ and treated this parameter as a known constant for the purpose of calculating the second derivatives of the log likelihood.

our models (models 1 and 2), the LR test statistic for the same hypothesis is as high as 3.44, rejecting the null hypothesis at a 10 percent, if not 5 percent, significance level.

Note that the transitory component of real GDP is subject to two different shocks: an asymmetric, discrete, shock π_{S_t} and a symmetric, continuous, shock u_t. The parameters σ_{u0} and σ_{u1} measure the relative importance of the symmetric shock. Focusing our discussion on the models with asymmetry in the transitory components, comparison of models 1 and 3 leads us to a test of the joint hypothesis that $\sigma_{u0} = \sigma_{u1} = 0$. We get the LR statistic of 0.50, accepting the hypothesis with a very high p-value. The test result suggests that the discrete shock π_{S_t} explains most of the dynamics in the transitory component. During the normal times, the economy is subject mostly to permanent shocks and it is operating near the trend ceiling. During the recession times and the recovery periods that follow, however, the transitory component plays a major role in the output fluctuations.[5]

Once a series of negative transitory shocks hit the economy or plucks output down, their effects decay relatively fast as implied by $\phi_1 + \phi_2$, that is, these negative shocks are relatively short-lived. Near the end of a recession, with no further new negative shocks, the fast-decaying negative shocks give rise to a third, high recovery, phase. By the time the effects of these negative shocks are all gone, the economy is operating near the trend ceiling again. Thus, our model gives rise to three, instead of two, dis-

5. We also estimated a more general model in which we allowed recessions to have both permanent and transitory effects. This was done by replacing v_t in equation (10) by $\theta S_t + v_t$ with $\theta \neq 0$. The estimate of θ was insignificant.

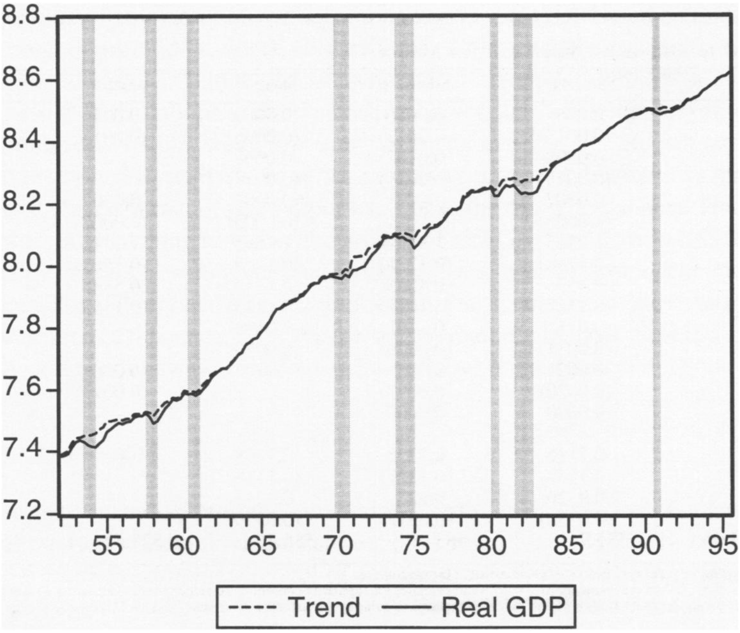

FIG. 1. Real GDP and Its Trend Component

tinctive phases of the business cycle dynamics, as implied by Sichel (1994): a normal phase, a recessionary phase, and a high growth, recovery phase.

Figures 1 through 4 visually summarize the preceding discussions. These are filtered estimates, $\tau_{t|t}$ and $c_{t|t}$, rather than smoothed estimates, $\tau_{t|T}$ and $c_{t|T}$, because obtaining the latter is problematical due to the nonlinearity of the Markov-switching model.[6] Our interpretation of the results, however, would not be altered at all. Figure 1 depicts the log of real GDP (y_t) and estimates of its trend component or the trend ceiling component ($\tau_{t|t}$): most of the time, the economy is operating on or near the trend ceiling component; during the NBER recession periods as represented by the shaded area and the recovery periods that follow, the economy is operating below the trend ceiling component. Figures 2 and 3 describe this point more clearly. The two figures depict estimates the transitory component ($c_{t|t}$) and the probability of a negative shock ($Pr[S_t = 1|\psi_t]$) to the transitory component. We clearly observe periods of

6. The smoothing algorithm for a state-space model with Markov switching is not as straightforward as that for a linear state-space model. As the Appendix describes, the filtered estimates of the unobserved components involve approximations. Such approximations are necessary to make the Kalman filter operable. In order to get smoothed estimates, one needs additional approximations as Kim (1994) has shown. Thus, even though the smoothed estimates are obtained using more information than the filtered estimates, they also involve successive approximations. For a discussion of the smoothing algorithm in the context of a general non-Gaussian state-space model within the classical framework, refer to Kitagawa (1987). The Bayesian Gibbs-sampling approach to state-space models with Markov switching by Kim and Nelson (1998a, 1998b) provides an alternative approach to smoothing but is subject to Monte Carlo sampling error and limitations related to the speed of convergence.

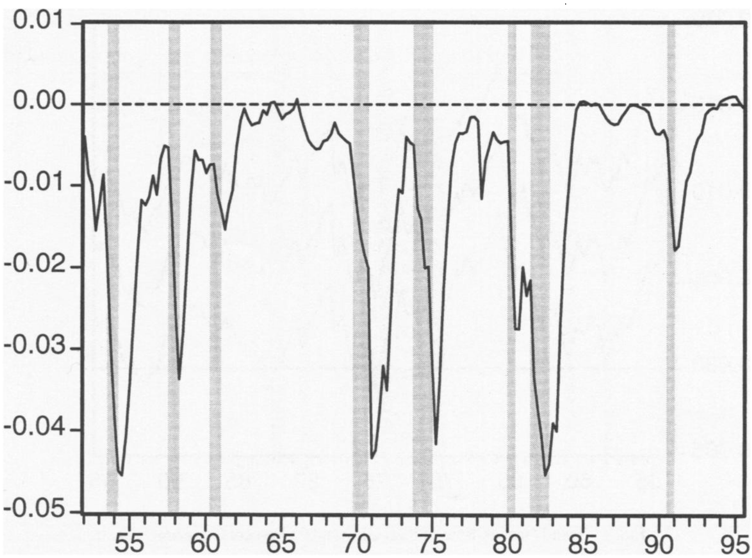

FIG. 2. Transitory Component of Real GDP

decreasing transitory component or periods of high probabilities of negative shocks that are highly correlated with the NBER recessionary periods. In Figure 2, during the short periods right after the NBER recessionary periods, we can observe how the negative transitory shocks are deteriorating, restoring the the economy back to the trend ceiling, or the normal level. Figure 4 depicts the estimates of the trend growth term

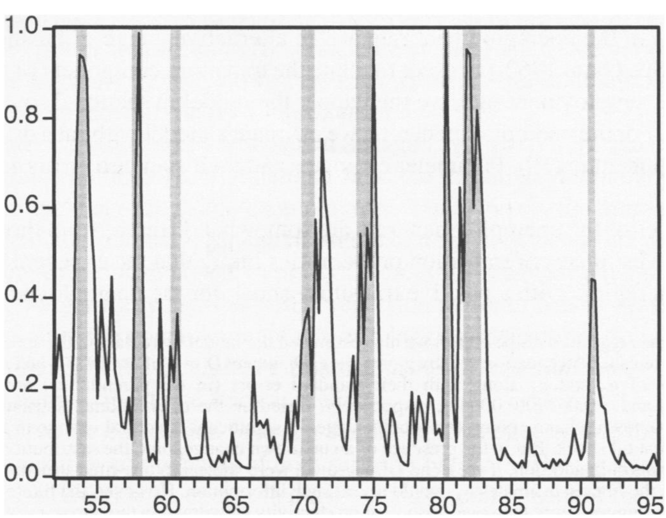

FIG. 3. Probabilities of Negative Shocks to Transitory Component of Real GDP

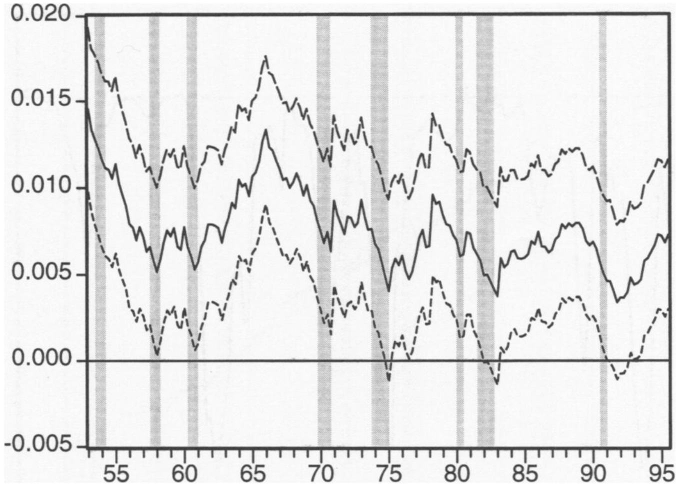

FIG. 4. Trend Growth Rate of Real GDP and 95 Percent Confidence Bands

($g_{t|t}$), along with the 95 percent confidence bands. There are periods in which the confidence bands do not overlap (early 1950s or mid-1960s versus early 1990s), suggesting the significance of a productivity slowdown in the postwar GDP series. Along with earlier test results on the significance of σ_w, this further justifies the specification of the stochastic trend growth term in equations (11)–(12).[7]

3.2 Dynamics of the Unemployment Rate and Okun's Law

Fluctuations in the unemployment rate are an alternative gauge of business cycle fluctuations. (See Okun 1962.) In order to relate the transitory component of real GDP to that of the unemployment rate, we then apply the model in section 2 to the unemployment rate. For the unemployment rate, we estimate a model without a drift term in the trend component in (10). Parameter estimates and their standard errors are reported in Table 2.[8]

The dynamics of the unemployment rate are somewhat different from those of real GDP. First, the estimates of transition probabilities imply that the expected duration, $(1/1 - p)$, of a regime with a positive transitory shock for the unemployment rate is

7. Following the suggestion of the editor, we also estimated the model by replacing the specification of the growth rate of real GDP in equation (11) by $g_t = g_0 + g_1 D_t$, where $D_t = 1$, if $t > 1973.1$ and $D_t = 0$, otherwise. Estimates of g_0 and g_1 along with their standard errors (in the parentheses) are given by: 0.00868(0.00031) and −0.00290(0.00047), respectively. Based on the conventional distribution theory with an assumed (known) change point, this would suggest a significant structural change in the average growth after the first oil shock. But in the presence of an unknown changepoint, the distribution of the test statistic is nonstandard. In addition, if the trend GDP growth were subject to one-time structural break toward a lower rate, the filtered estimates $g_{t|t}$ would reveal such information. However, the filtered estimates in Figure 4 seem to suggest more than one episode of productivity slowdown in the postwar sample.

8. When Clark's (1987) linear unobserved components model was applied to the unemployment rate, parameter estimates failed to converge.

TABLE 2

MAXIMUM LIKELIHOOD ESTIMATION OF THE MODEL: UNEMPLOYMENT RATE

Parameters	Model 1	Model 2
σ_{v0}	0.0006	0.0020
	(0.0005)[a]	(0.0002)
σ_{v1}	0.0017	0.0035
	(0.0007)	(0.0006)
ϕ_1	1.4911	1.3801
	(0.0897)	(0.0753)
ϕ_2	-0.5639	-0.5160
	(0.0810)	(0.0697)
π	0.0016	0.0637
	(0.0008)	(0.0009)
σ_{u0}	0.0014	—
	(0.0003)	
σ_{u1}	0.0040	—
	(0.0005)	
p	0.9122	0.5122
	(0.0506)	(0.1828)
q	0.9529	0.9038
	(0.0256)	(0.0316)
Log Likelihood	790.71	776.90

[a]Standard errors of the parameter estimates are reported in the parentheses.

longer than that of a regime with a negative transitory shock for real GDP. Second, the sum of the AR coefficients ($\phi_1 + \phi_2 = 0.9272$) of the transitory component is much higher for the unemployment rate than for real GDP. This suggests that, after the economy is hit by a series of negative shocks to real GDP and positive shocks to the unemployment rate, it takes more time for the unemployment rate to go back to its normal rate or the trend unemployment rate. Third, the symmetric shock to the transitory component, u_t, seems to be more important than in the case of real GDP. Comparing the log likelihood values for models 1 and 2 in Table 2, we very strongly reject the hypothesis that $\sigma_{u0} = \sigma_{u1} = 0$. This suggests an existence of periods in which the unemployment rate falls below its trend level.

In Figure 5, the unemployment rate is depicted against its trend rate. The trend rate seems to have increased until the mid-1980s, but this upward trend does not seem to be dominating since then. The unemployment rate is below the trend rate in the late 60s, the late 80s, and the mid 90s. In Figure 6, deviations of the unemployment rate from its trend rate are depicted. Periods of increasing transitory unemployment correspond to, but generally lag, the NBER recession periods. This is consistent with the classification of the unemployment rate as a lagging indicator at troughs, though not with the finding of the Conference Board (*Business Cycle Indicators,* January 1997) that the unemployment rate leads at peaks.

To observe the correlation between the unemployment rate and the real GDP, we plot the transitory components of both series in Figure 7. The similarity between the two series is remarkable, except that the transitory component of the unemployment rate is somewhat lagging and more persistent. Correlation between the two transitory components is estimated to be -0.6938.

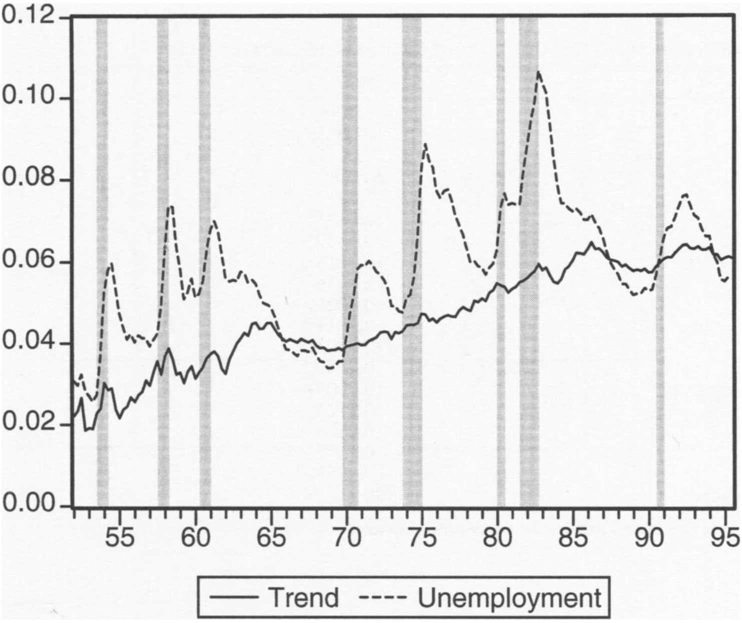

FIG. 5. Unemployment Rate and Its Trend Component

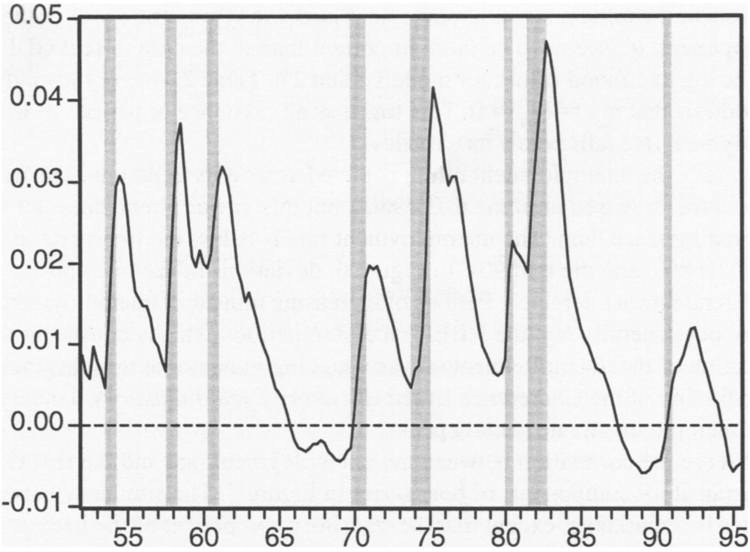

FIG. 6. Transitory Component of Unemployment Rate

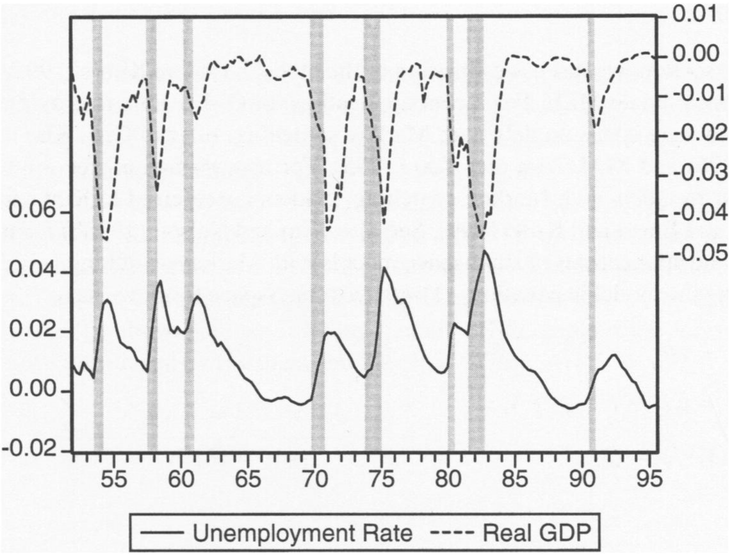

FIG. 7. Transitory Components: Real GDP and Unemployment Rate

4. SUMMARY AND CONCLUSION

An econometric time series model that incorporates features of Friedman's (1964, 1993) "plucking" model of business fluctuations is presented. What distinguishes our model from other existing nonlinear models of the business cycle is asymmetry in the transitory components of measures of business fluctuations. The model also allows us to test for the existence of a trend ceiling component for real GDP or a trend floor component for the unemployment rate. For quarterly real GDP, we cannot reject the hypothesis that the stochastic trend, or the potential real GDP, provide the upper limit to the output set by the available resources and methods of organizing them. For the quarterly unemployment rate, however, estimates of the trend rate or the trend rate does not seem to provide the lower limit. The transitory dynamics of the unemployment rate seems to be almost a mirror image of the transitory dynamics of real GDP, with slight lags and higher persistence for the unemployment rate.

Empirical results for real GDP suggest that output during normal times is driven mostly by permanent shocks: Real business cycle models may be more relevant in explaining the output dynamics during normal times. During the recessionary and high-growth recovery periods, real GDP is driven mostly by transitory shocks: Macroeconomic theories such as monetary models or other models that emphasize demand-oriented shocks may be more appropriate.

APPENDIX

In this section, we discuss estimation of the model based on Kim's (1993a, 1993b, 1994) approximate MLE. For recent applications of C.-J. Kim's approximate MLE method to state-space models with Markov switching, refer to C.-J. Kim and M.-J. Kim (1996) and M.-J. Kim and Yoo (1995). For approximation-free inferences of state-space models with Markov switching, readers are referred to Kim and Nelson (1998a) and Engel and Kim (1999). See also Kim and Nelson (1998b) for more discussion and applications of state-space models with Markov switching.

Writing the model in equations (1)–(9) in a state-space form, we have

$$y_t = H\xi_t \tag{A.1}$$

$$\xi_t = \tilde{\mu}_{S_t} + F\xi_{t-1} + V_t\,, \tag{A.2}$$

$$E(V_t V_t') = Q_{S_t}\,, \tag{A.3}$$

where

$$H' = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix},\ \xi_t = \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ g_t \end{bmatrix},\ \tilde{\mu}_{S_t} = \begin{bmatrix} 0 \\ \pi_{S_t} \\ 0 \\ 0 \end{bmatrix},\ F = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \phi_1 & \phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ V_t = \begin{bmatrix} v_t \\ u_t \\ 0 \\ w_t \end{bmatrix},$$

and

$$Q_{S_t} = \begin{bmatrix} \sigma^2_{v,S_t} & 0 & 0 & 0 \\ 0 & \sigma^2_{u,S_t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_w \end{bmatrix}$$

Then, conditional on $S_t = j$ and $S_{t-1} = i$, the Kalman filter algorithm can be written as follows:

$$\xi^{(i,j)}_{t|t-1} = \tilde{\mu}_j + F\xi^{i}_{t-1|t-1}\,, \tag{A.4}$$

$$P^{(i,j)}_{t|t-1} = FP^{i}_{t-1|t-1}F' + Q_j\,, \tag{A.5}$$

$$\eta^{(i,j)}_{t|t-1} = y_t - H\xi^{(i,j)}_{t|t-1}\,, \tag{A.6}$$

$$f^{(i,j)}_{t|t-1} = HP^{(i,j)}_{t|t-1}H'\,, \tag{A.7}$$

$$\xi^{(i,j)}_{t|t} = \xi^{(i,j)}_{t|t-1} + P^{(i,j)}_{t|t-1}H'\left[f^{(i,j)}_{t|t-1}\right]^{-1}\eta^{(i,j)}_{t|t-1}\,, \tag{A.8}$$

$$P_{t|t}^{(i,j)} = \left(I - P_{t|t-1}^{(i,j)} H' \left[f_{t|t-1}^{(i,j)}\right]^{-1}\right) H P_{t|t-1}^{(i,j)}, \tag{A.9}$$

where $\xi_{t|t-1}^{(i,j)}$ is an inference about ξ_t based on information up to time $t-1$; $\xi_{t|t}^{(i,j)}$ is an inference about ξ_t based on information up to time t; $P_{t|t-1}^{(i,j)}$ and $P_{t|t}^{(i,j)}$ are the MSE matrices of $\xi_{t|t-1}^{(i,j)}$ and $\xi_{t|t-1}^{(i,j)}$, respectively; $\eta_{t|t-1}^{(i,j)}$ is the conditional forecast error of y_t based on information up to time $t-1$; $f_{t|t-1}^{(i,j)}$ is the variance of $\eta_{t|t-1}^{(i,j)}$.

As noted by Gordon and Smith (1988) and Harrison and Stevens (1976), each iteration of the above Kalman filtering produces a two-fold increase in the number of cases to consider. Therefore, at the end of each iteration of the Kalman filter, we need to collapse the (2 × 2) posteriors ($\xi_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$) into two by employing approximations, to make the above Kalman filtering operable. Kim employs the following approximations introduced by Harrison and Stevens (1976):

$$\xi_{t|t}^{j} = \frac{\sum_{i=1}^{2} Pr[S_{t-1} = i,\ S_t = j|\psi]\,\xi_{t|t}^{(i,j)}}{Pr[S_t = j|\psi]}, \tag{A.10}$$

and

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{2} Pr[S_{t-1} = i,\ S_t = j|\psi_t]\{P_{t|t}^{(i,j)} + (\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)})(\xi_{t|t}^{j} - \xi_{t|t}^{(i,j)})'\}}{Pr[S_t = j|\psi_t]}, \tag{A.11}$$

where ψ_t refers to information available at time t.

At each iteration, given $S_t = j$ and $S_{t-1} = i$, we have the conditional forecast error ($\eta_{t|t-1}^{(i,j)}$) and its variance ($f_{t|t-1}^{(i,j)}$), which will be used to calculate the conditional density of y_t. Considering the fact that S_t and S_{t-1} are unobserved, we can calculate the conditional density of y_t in the following way:

$$\begin{aligned} f(y_t|\psi_{t-1}) &= \sum_{j=0}^{1}\sum_{i=0}^{1} f(y_t, S_t = j,\ S_{t-1} = i|\psi_{t-1}) \\ &= \sum_{j=0}^{1}\sum_{i=0}^{1} f(y_t|S_t = j, S_{t-1} = i, \psi_{t-1})\,Pr[S_t = j, S_{t-1} = i|\psi_{t-1}], \end{aligned} \tag{A.12}$$

where

$$f(y_t|S_{t-1} = i,\ S_t = j,\ \psi_{t-1}) = \frac{1}{\sqrt{2\pi f_{t|t-1}^{(i,j)}}}\,exp\left(-\frac{1}{2}\,\frac{\eta_{t|t-1}^{(i,j)\,2}}{f_{t|t-1}^{(i,j)}}\right), \tag{A.13}$$

and

$$Pr[S_t = j, S_{t-1} = i|\psi_{t-1}] = Pr[S_t = j|S_{t-1} = i]Pr[S_{t-1} = i|\psi_{t-1}], \tag{A.14}$$

where $Pr[S_t = j|S_{t-1} = i]$, $i = 0, 1, j = 0, 1$, are given by the transition probabilities.

The last thing that remains is to calculate $Pr[S_t = j|\psi_t]$, which is given as follows:

$$Pr\{S_t = j|\psi_t] = \sum_{i=0}^{1} Pr[S_t = j, S_{t-1} = i|\psi_t], \tag{A.15}$$

where

$$\begin{aligned} Pr[S_t = j, S_{t-1} = i|\psi_t] &= Pr[S_t = j, S_{t-1} = i|\psi_{t-1}, y_t] \\ &= \frac{f(y_t, S_{t-1} = i, S_t = j|\psi_{t-1})}{f(y_t|\psi_{t-1})} \\ &= \frac{f(y_t|S_t = j, S_{t-1} = i, \psi_{t-1}) Pr[S_t = j, S_{t-1} = i|\psi_{t-1}]}{f(y_t|\psi_{t-1})}. \end{aligned} \tag{A.16}$$

To start the filter, we use the steady-state probabilities given by

$$Pr[S_0 = 0|\psi_0] = \frac{1-p}{2-p-q} \text{ and } Pr[S_0 = 1|\psi_0] = \frac{1-p}{2-p-q}. \tag{A.17}$$

As a by-product of running the above filter, the conditional log likelihood function can be obtained from equation (A.12). The sample conditional log likelihood is

$$LL = \sum_{t=1}^{T} \log(f(y_t|\psi_{t-1})), \tag{A.18}$$

which can be maximized with respect to unknown hyperparameters of the model.

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